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Volcanic eruption plume top topography and heights as determined from photoclinometric analysis of satellite data

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Abstract. Photoclinometry, a shape-from-shading technique, is applied to satellite data to determine the three-dimensional height and morphology of the upper optical surface of a plume from the eruption of Redoubt Volcano, Alaska. The technique can be applied to visible images of volcanic plumes that have uniform scattering properties and is most effective on images with moderate incident Sun angles and plume transport perpendicular to the azimuth of the Sun. The two most significant sources of error are the finite resolution of the image data and the choice of image data number (DN) value for a flat plume element. Provided the plume spans at least 10 image pixels, useful results can be obtained for the central portion of the plume. The closest approximation to a flat plume element is found to be the visible DN value corresponding to the coldest pixel along a transect. Errors in the element altitude due to uncertainty in the DN value for a flat pixel amount to ~10–20 m per pixel and are cumulative along the plume transect. For Redoubt this results in an uncertainty of approximately 300 m at the highest points. The method indicates that the Redoubt plume rose to an altitude of approximately 3 km while traveling a distance of 150 km from the vent and that the surface topography of the plume exhibits influences of the ambient wind through simple wave structures. This technique will have a tremendous impact on studies of plume spreading dynamics and the time-integrated evolution of plumes.

1. Introduction

Volcanic eruptions can produce plumes that rise more than 40 km above the Earth's surface [*Holasek et al.*, 1996a], deposit ash over tens of thousands of square kilometers [*Sparks*, 1986], and disperse gases and aerosols over hemispheric scales [*McCormick et al.*, 1995]. In addition to the dangers posed by ash deposition in populated areas, airborne ash can also be a hazard because of the potential for its ingestion into aircraft engines and their resultant failure [*Casadevall*, 1994].

Progressively more detailed theoretical models of plume rise [*Morton et al.*, 1956; *Wilson*, 1976; *Wilson et al.*, 1978; *Sparks*, 1986; *Wilson and Walker*, 1987; *Woods*, 1988; *Glaze et al.*, 1997] have shown that the development of eruption plumes is generally a complex process. Despite evidence that the ash and gas components of some plumes may separate, ultimately rising to different altitudes [*Holasek et al.*, 1996b; *Seftor et al.*, 1997], all of these models assume well-mixed plumes where the ash and gas rise confluently. The models describe plumes that initially spread both laterally and vertically at speeds determined by the mass eruption rate of magma from the vent, the exsolved gas content of the erupting magma, and the local vertical wind speed and humidity profiles. A major outcome of the early modeling work was the demonstration that the maximum height to which a plume will rise is determined almost completely by the mass eruption rate of magma from the vent [*Settle*, 1978; *Wilson et al.*, 1978]; it is the single most important parameter controlling the dispersal of particles, aerosols and gases and is the most useful quantity to know in any investigation of a plume's eruptive history and likely future behavior. Conversely, knowledge of the height of an eruption plume provides important information on the magma eruption rate.

Once an eruption column has reached its maximum height, the plume material may behave as a gravity current [*Bursik et al.*, 1992] seeking its neutral buoyancy level, and subsequently be transported laterally by the prevailing winds [*Armienti et al.*, 1988; *Glaze and Self*, 1991]. The morphology of the upper surface of the volcanic plume can provide clues needed to better understand the dynamics of the gravity current behavior as well as the influence of the ambient atmosphere on the plume transport. For example, the 1991 Mount Pinatubo eruption plume exhibited large concentric waves on its upper surface [*Holasek et al.*, 1996a] that formed as the enormous umbrella portion of the plume spread laterally. Space-based images are the only way to observe the morphology of the upper surface of these large plumes, as they provide unique information needed to refine and test existing theoretical models of plume rise, spread, and transport.

The objective of this paper is to develop a method by which existing satellite systems, notably the advanced very high resolution radiometer (AVHRR), can be used to determine the

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Figure 1. (a) Vertical cross section of an eruption plume showing material overshooting the neutral buoyancy height due to its inertia (region labeled A) and consequently becoming undercooled (colder than the ambient atmosphere). The temperature increases (region B) as material is transported laterally and descends again to the neutral buoyancy level (NBL). (b) Variations in the appearance of an eruption plume from a polar-orbiting satellite as a function of the angle relative to the nadir at which the plume is viewed. View 1 is representative of a nadir view. Views 2 and 3 are viewed at progressively larger off-nadir angles.

height and morphology of an eruption plume. This information can then be used to infer eruption conditions as well as to aid in understanding the relative influence of various transport mechanisms. In contrast to previous studies that take a single estimate of the height of a plume as its characteristic height at a specific time, we present a method for determining the spatial distribution of the plume height. This involves a new application of the technique called "photoclinometry," a shape-from-shading technique that interprets pixel-to-pixel brightness variations as changes in the inclination to the local normal of the surface elements. The photoclinometric technique has been applied mainly to images of the surfaces of other planets [*Bonner and Schmall*, 1973; *Mouginis-Mark and Wilson*, 1981; *Davis and Soderblom*, 1984] but is particularly well suited to the analysis of daytime AVHRR images of plumes. This technique can determine the three-dimensional height distribution of the upper optical surface of the visible (primarily ash and ice) plume over a large fraction of its entire spatial extent. Thus plume heights generated by this method provide relative morphologic information for very large umbrella clouds as well as data on the "integrated evolution" of plumes transported by the wind, from the present conditions (the part of the plume directly over the vent) to downwind plume material that may be many hours old.

We begin with a summary of the physical properties of eruption plumes and the ways in which their morphologies affect methods of obtaining their heights. We then briefly review the limitations of existing methods for determining eruption plume heights. The photoclinometric technique is described and demonstrated using an AVHRR image of the December 16, 1989, eruption plume from Redoubt volcano, Alaska. The sources of systematic error are then discussed and the implications for the resulting plume shape are presented. We close with a brief discussion of the future uses of space-borne systems for the collection of plume topography data and for validation of the photoclinometry technique.

2. Imaging of Eruption Plumes

Volcanic eruption plumes can adopt a range of shapes, the details of which depend on whether the source is localized (e.g., forming a Plinian eruption plume [*Sparks*, 1986]) or distributed (e.g., when a pyroclastic flow deposit leads to the production of a co-ignimbrite eruption plume [*Self and Walker*, 1994]). Point source plumes are typically strongly influenced by their interaction with the ambient atmosphere (Figure 1a). The wind speed profile leads to a deflection of the plume during ascent and results in a distinctive profile of the top of the plume when viewed perpendicular to the dominant wind direction. The plume also exhibits a distinctive thermal anomaly at its highest point because the plume material overshoots its neutral buoyancy level (indicated by NBL in Figure 1a), cooling adiabatically as it does so, and then increases in temperature as it recompresses on descending to a lower level [*Woods and Self*, 1992].

In general, plumes generated by localized vents and distributed sources are indistinguishable from one another in satellite images. The appearance of an eruption plume in an image obtained by a satellite is more strongly dependent on the angle, measured from nadir, at which the plume is viewed. Figure 1b illustrates the viewing scenarios that may arise from polar-orbiting weather satellites with broad swath widths (many hundreds of kilometers). The typical shapes of eruption plumes are such that at small off-nadir angles (view 1 in Figure 1b), only the upper convex surface of a plume will be imaged in plan view. No detailed information is available about the lower parts of the plume, where most of its rise and lateral expansion is taking place, but a large fraction of the upper surface will be both illuminated and visible. For much larger off-nadir angles (which may be up to $\sim$50° in the extreme cross-track parts of AVHRR scenes [*Holben and Fraser*, 1984]), the lower sides and edges of plumes may also be visible (views 2 and 3 in Figure 1b). They will, however, be greatly foreshortened, as will the upper surfaces of the plumes, reducing the effective resolution of the images, as has been discussed by *Holasek and Self* [1995] for GOES images of the 1980 Mount St. Helens eruption. Imaging systems with extreme along-track or cross-track capabilities would be able to obtain elevation views of plumes, but only at relatively low resolutions.

For this study we have used an AVHRR image of a plume acquired during the December 1989 eruption of Redoubt volcano in Alaska (Figure 2). This plume was generated by the continuous low-level emission of volcanic ash over several hours, and its shape is indicative of transport by dominant ambient winds (similar to that shown in Figure 1a). The AVHRR instruments fly on the operational NOAA weather satellites in polar orbits and image the Earth at a nominal nadir resolution of $\sim$1.1 km pixel^{-1}. Each NOAA satellite has a repeat viewing cycle of 12 hours. At high latitudes, areas are seen more frequently owing to the overlap of data coverage (as many as 12 scenes per day for two operating satellites). Data are obtained at five wavelengths: band 1, visible (0.58–0.68 μm); band 2, near-infrared (0.72–1.10 μm); band 3, shortwave infrared (3.55–3.93 μm); and bands 4 and 5, two longwave or thermal infrared (10.3–11.3 and 11.5–12.5 μm) channels. The AVHRR data set is selected for this study specifically because

Figure 2. NOAA 11 AVHRR image showing an eruption plume from Redoubt volcano, Alaska, at 2337 UT on December 18, 1989. (a) Simple linear stretch of the band 1 visible data. (b) Linear stretch of the negative band1/band2 ratio, included here to facilitate the identification of the plume (P) and the shadow (S). The coastline of Cook Inlet is shown in both images. In Figure 2a, Redoubt volcano is marked by an asterisk, the city of Anchorage is marked by an "A," and Kenai is shown by a "K." The black rectangle in Figure 2b delineates the area within which the photometric studies were conducted and corresponds to rows 460–570 and columns 560–700.

it is available now and because the wide field of view of the sensor (2700-km swath width) leads to a high probability of observing transient eruption plumes. This data set also serves as an analog for future satellite instruments, such as the Moderate-Resolution Imaging Spectroradiometer (MODIS) that will fly on several of the NASA Earth Observing System (EOS) platforms in the 1998 to 2010 time frame. This new system will have a swath width in excess of 1400 km and will have a spatial resolution of ~250 to 1000 m in the visible and infrared wavelength regions [*Ardanuy et al*., 1991].

3. Methods of Deducing Eruption Plume Height and Topography

A variety of techniques have been used to determine the heights of eruption plumes. Although ground-based and air-

Figure 3. Positions of the root (R) and foot (F) of the shadow extending from an eruption plume to the ground shown for high and low solar incidence angles.

borne observations can be used to estimate the heights of small plumes [e.g., *Brantley*, 1990], the synoptic view provided by satellite images is more appropriate for larger Plinian plumes and for plumes caused by mixing between atmospheric gas and actively moving pyroclastic flows (co-ignimbrite plumes) owing to their large areal extents [*Sparks et al*., 1986].

A seemingly obvious way to investigate the altitude and vertical structure of a plume would be to study stereo satellite images of it. Unfortunately, to our knowledge, there is only one instrument currently flying, the Along Track Scanning Radiometer (ATSR) on the European ERS-2 satellite, that can provide stereo images of transient phenomena. While ATSR stereo data have been used to determine meteorologic cloud heights [*Prata and Turner*, 1997], to date, no ATSR observations of volcanic plumes have been reported in the open literature. Although the French SPOT satellite does have the ability to obtain stereo data, this instrument produces stereo pairs by viewing in the cross-track direction at different angles [*Chevrel et al*., 1981]. Thus the stereo model has to be developed from data collected on different orbits, which might be separated in time by a few days or even months. While this is acceptable for mapping fixed targets on the ground, the shape and structure of a transient eruption plume will change on a timescale of minutes, precluding any possibility of stereo fusion. The Optical Sensor (OPS) on the Japanese JERS-1 spacecraft provides along-track stereo; however, the duty cycle of the instrument is such that it is not operating continuously, and mission operations require targets to be defined several weeks in advance, precluding the imaging of "unexpected" transient phenomena. Much shorter timescale stereo coverage will eventually be available from the along-track cameras of the MISR and ASTER systems, and we discuss the future use of these data in section 7.

An accepted method for determining plume heights from satellite data has been to use their temperatures measured in the thermal infrared [e.g., *Kienle and Shaw*, 1979; *Glaze et al*., 1989; *Holasek et al*., 1996a]. This method assumes that the ash-laden plume has an emissivity close to unity, which is yet to be verified, and that the plume top and the ambient air are in equilibrium (i.e., at the same temperature). While this approach seems to produce reasonable plume heights for some volcanic plumes and could, in theory, be used to generate plume top topography, there are many shortcomings. First, in order for this technique to work at all, an atmospheric temperature profile must be available for the region of interest. Also, owing to the nature of the atmospheric temperature gradient (which is typically negative from the surface to the tropopause and zero or positive in the stratosphere), a given temperature might correspond to more than one altitude in the profile data. While this difficulty can sometimes be overcome by comparison of plume trajectories with wind speeds and directions, there are often times when the atmosphere displays nonuniform changes in temperature and wind velocity over a wide range of altitudes. Finally, it is important to be aware that plumes can dynamically overshoot their level of neutral buoyancy and thus become significantly undercooled over the vent [*Woods and Self*, 1992], violating the initial assumption of ambient equilibrium.

Another method that has been used with relative success is the shadow technique. Shadow lengths measured from Geostationary Operational Environmental Satellite (GOES) images have been used by *Glaze et al*. [1989] and *Holasek and Self* [1995] to calculate the heights of plumes from Lascar and Mount St. Helens volcanoes, respectively. During the day, all eruption plumes cast shadows onto the surface beneath them (either the ground or the tops of meteorologic clouds), and the length of that shadow can be measured from a visible channel of a satellite image. Knowledge of the elevation of the Sun in the region at the time the image was acquired allows this shadow length to be used to calculate the height of the shadow root above the underlying surface at the foot of the shadow (Figure 3). The problem with this technique, as Figure 3 demonstrates, is that the shadow forms a tangent to some point on the plume surface, and it is never possible to obtain a good estimate of the true height of the top of the plume unless the incidence angle of the Sun is close to 90°. However, at such high incidence angles the contrast between the shadow and the illuminated surface is minimal, and the penumbral part of the shadow is large, making the location of its foot poorly defined.

The lack of availability of stereoscopic images and the only partial relevance (due to the aforementioned geometric effects) of shadow-derived heights has been a common feature of planetary imaging for the last 20 years. It is often solved by the use of photoclinometry. We now discuss the application of photoclinometry to the analysis of topography in general, and especially that of eruption plumes.

4. Topography Determination Using Photoclinometry

4.1. Photometric Properties of Surfaces

The relationship between the apparent brightness (more specifically the bidirectional reflectance) of a surface element and its orientation in space relative to the observer and the illumination source is quite complex. The most general expression for the bidirectional reflectance in the visible part of the spectrum can be derived from radiative transfer theory (see *Hapke* [1986] and references therein to earlier papers by Hapke). According to this expression, the energy flux F reaching a detector can be expressed as a function of three angles and a number of parameters defining the physical state of the scattering surface material:

$$F = Kw\left[\frac{\cos i}{\cos i + \cos e}\right] \Phi(\psi, i, e, w, b, c, \Theta). \quad (1)$$

In equation (1), K is a constant taking account of the intensity and distance of the light source from the target and of the distance of the observer from the target; w is the single scattering albedo of the target surface; i is the incidence angle (the angle between the local normal to the surface at the point being observed and the direction to the source of illumination,

Figure 4. Geometries for the incidence and emergence angles for (a) a horizontal surface element (i and e) and (b) a surface element tilted through the angles t and ϕ (i_t and e_t). The incidence and emergence angles in both cases are measured from the normal to the surface element. The x direction indicates the scan direction (rows or columns) that is most nearly parallel to the direction of solar incidence, and the angle A is defined as the azimuthal bearing of the Sun from that scan direction. The tilt angle t is defined as lying in the x plane, and the cross-scan tilt ϕ is measured in the y plane.

in this case the Sun; see Figure 4a); e is the emergence angle (the angle between the normal to the target surface and the direction to the observer; see Figure 4a); ψ is the angle of vision (the total angle between the vectors from the point observed to the illumination source and the observer); b and c are parameters expressing the relative amounts of forward, sideways, and backward scattering from the smallest constituent particles of the materials making up the target surface; Θ is a parameter characterizing the root-mean-square roughness of the target surface integrated over all subpixel scales [*Efford*, 1991]; and Φ is a complicated function of i, e, ψ, w, b, c, and Θ (all notation is defined in the notation section following the main text).

Fortunately, over that part of a satellite scene containing the image of an eruption plume, the variation in distance from the target surface to the spacecraft detector will almost always be small enough that the emergence angle will be essentially the same for all of the image pixels involved (the rare instances where this is not true involve eruption plumes the size of the June 15, 1991, Pinatubo plume, which extended over more than 20% of an AVHRR scene [*Holasek et al.*, 1996a]. This enormously simplifies the use of equation (1), for the value of

Figure 5. Relationship between the tilt angle t of a surface element from the horizontal and the brightness ratio B_r for (a) solar incidence angles $i = 10°$, $40°$, and $70°$ when the emergence angle e is $0°$, i.e., the plume is close to the nadir and (b) emergence angles $e = 10°$, $40°$, and $70°$ (i.e., the plume is off nadir by these amounts) for a solar incidence angle $i = 45°$.

Φ is then a constant, which may conveniently be absorbed into the factor K. We can also absorb into K both the calibration factor which converts energy flux into image data numbers (DNs) and the single-scattering albedo w (provided for the moment that there are no significant variations in w over the part of the image of interest; we return to this important point later). We can then write (1) explicitly for a pixel representing a surface element that is exactly flat, i.e. for which the angle of tilt, t, from the horizontal is exactly zero (see Figure 4a):

$$\mathrm{DN}(i, e, t = 0) = K\left[\frac{\cos i}{\cos i + \cos e}\right], \quad (2)$$

where DN is an integer (usually between 0 and 255) representing the amount of radiant energy received at the sensor for a given pixel. It should be noted that equation (2) makes physical sense in the limit as i goes to 90° or e goes to 0°.

4.2. Influence of Viewing Geometry

We now consider the consequences of the target element being tilted through a finite angle from the horizontal. Because we intend to use these tilt angles to measure elevation changes across the image between points on the surface represented by the edges of the pixels, it is convenient to define the tilt angle t to be measured in the vertical plane that is parallel to either the rows or the columns of the pixels forming the image. The maximum sensitivity is obtained when the plane containing the solar incidence angle is as close as possible to the plane in which the tilts are measured [*Efford*, 1991]. For an AVHRR image the rows of the image are oriented roughly east-west and the columns are oriented north-south. Thus for a near-equatorial image the solar incidence plane will be roughly east-west, and so topographic scans would be best extracted using rows of DNs. Conversely, for a near-polar image the solar incidence plane will be more nearly north-south than east-west, and topographic scans would be best extracted using columns of DNs. The tilt angle t is then defined as lying in the plane of whichever scan direction (rows or columns) is being used (Figure 4b), and the angle A is defined as the azimuthal bearing of the Sun from the scan direction (Figure 4a). The angle ϕ is the angle of tilt in the cross-scan direction. For the purposes of this study we have assumed that $\phi = 0$. Using these definitions, it is a matter of simple spherical trigonometry to show that the DN observed for a tilted surface element will be

$$\mathrm{DN}(i, e, t) = K\left[\frac{\cos i_t}{\cos i_t + \cos e_t}\right], \quad (3)$$

where

$$\cos e_t = \cos e \cos t, \quad (4)$$

$$\cos i_t = \cos i \cos t + \sin i \sin t \cos A. \quad (5)$$

For a fully calibrated satellite image, all of the geometric and radiance-related factors collected into the constant K in equations (2) and (3) will be known except for the inherent reflectivity (single scattering albedo w) of the target materials. Because we are intending to analyze a single image without recourse to any kind of stereoscopic information, we have no explicit way of separating the effects on pixel DNs of variations in w from the effects of variations in t. We get around this problem by assuming that we can always estimate the DNs of some pixels in the scene which represent materials that are locally flat, i.e. that we know the value of $\mathrm{DN}(i, e, t = 0)$ in (2). How this can be accomplished will be described shortly. If we then assume that the surface of interest is uniform in composition (i.e., pixel-to-pixel variations in w are negligible), the value of K will be the same for all pixels (regardless of tilt). Note that it is not important to know the composition of the material, as long as it is uniform in the region of interest (e.g., a mixture of volcanic ash, water vapor, and ice). We can then define the brightness ratio for each pixel in the part of the scene of interest (in this case the upper surface of the eruption plume) as the ratio of the DN observed for that pixel divided by the DN of a horizontal plume element:

$$B_r = \frac{\mathrm{DN}(i, e, t)}{\mathrm{DN}(i, e, t = 0)}. \quad (6)$$

Substituting equations (2) and (3) into (6) and solving for t, we find that

$$\tan t = \frac{(1 - B_r)(\cos i + \cos e) \cos i}{\sin i \cos A[(B_r - 1) \cos i - \cos e]}. \quad (7)$$

Note that (7) has a singularity at $i = 0°$ because topographic variations on a surface having scattering properties given by (1) do not produce brightness variations when $i = 0°$. The nature of the relationship when $i \neq 0°$ is shown in Figure 5. In Figure 5a, t is plotted as a function of B_r for three values of the solar incidence angle when the emergence angle is zero (i.e., the plume is close to the nadir) and $A = 23°$, a value appropriate to the image that we analyze in section 5. Note that for low incidence angles, estimated tilts are sensitive to very small changes in the brightness ratio. Conversely, for large incidence angles (Sun near the horizon), we quickly lose the ability to determine tilts for any pixels sloping more than a few degrees away from the Sun. In Figure 5b we show the consequences of e being nonzero (off-nadir viewing geometry) for the same value of A and an intermediate value of i $(= 45°)$. Figure 5b shows that, in comparison with a nadir view, an off-nadir viewing geometry most strongly affects surface elements tilted toward the Sun.

4.3. Generation of Topography

Once B_r has been defined for the ith image pixel of a given scan line and the corresponding value of t_i found from equation (7), the height change projected in the direction of the scan lines represented by this tilt, ΔH_i, is trivially found from

$$\Delta H_i = -\Delta X \tan t_i, \tag{8}$$

where ΔX is the distance on the ground represented by the width of each pixel. The absolute elevation of the location represented by the distal end of the ith pixel, measured from a datum defined by the proximal end of the first pixel, is then given by

$$H_j = \sum_{i=1}^{i=j} \Delta H_i. \tag{9}$$

To summarize, this section has shown that topographic profiles can be constructed along the rows or columns of an image as long as the image is photometrically calibrated (i.e., the DNs are linearly proportional to surface bidirectional reflectances) and as long as an estimate of the DN corresponding to a locally flat element of the plume top is available. We now describe the application of the technique to a volcanic plume in an AVHRR scene.

5. Application of Photoclinometry to a Volcanic Plume

5.1. Scene Calibration

Figure 2 shows part of a December 18, 1989, AVHRR image acquired at 2337 UT by the NOAA 11 satellite on orbit 6354. The image shows an eruption plume emanating from Redoubt volcano, Alaska, which was in almost continuous eruption from December 16 to 18 of that year [*Brantley*, 1990]. Both the bright plume (being blown to the east) and its shadow can readily be identified in this image. The Sun is shining roughly from the south (bottom edge) of the image in this high-latitude (~61°N) scene, causing the major systematic brightness variations due to the plume topography to occur along the north-south oriented columns of the image. The region analyzed in this study is delineated by the black box in Figure 2b.

The first matter to be considered in analyzing the image is that although the raw data numbers received from the satellite sensors have been calibrated (i.e., corrected for known nonlinearities in the response of the detectors), the DNs so derived are not exactly representative of surface reflectances. This is because the DNs contain a signal representing solar radiation scattered on to the ground by the atmosphere and then scattered again to the satellite. A correction is normally made for this multiple-scattered radiation by identifying any pixels that represent either a land surface situated within a meteorological cloud shadow or an area of water in which no specular reflection of the Sun occurs, and then subtracting the mean DN of such pixels from every DN in the image [*Lillesand and Kiefer*, 1987]. It can be seen in Figure 2 that two areas of water as well as the region representing the shadow of the eruption plume are contained in the image. The minimum DN value found in the areas containing water is 29, implying that this value should be subtracted from all DNs in the image as the basic correction for atmospheric multiple scattering. After correcting for the multiple scattering, we find that the mean DN value of pixels representing the region occupied by the plume shadow is 24. It is important to note that the mean DN value for the plume shadow is not zero, indicating that some of the sunlight incident on the plume penetrates through it or is scattered around it to reach the ground in its geometrical shadow. This apparent transparency of the plume has important implications for the application of the photoclinometry technique. Recall that we are able to eliminate the single scattering albedo w by assuming that the surface is uniform. If we are able to see varying amounts of the ground through the plume, there is no longer a surface of uniform composition. For the purposes of this study, a small degree of transparency does not impact the fundamental results; however, the technique's sensitivity to transparency is an important issue and will be addressed in subsequent work.

5.2. Plume Photoclinometry

In order to apply the photoclinometry technique, an appropriate DN value must be chosen for the locally flat elements of the plume and of the surrounding ground surface. Clearly, the simplest approximation that can be employed is to use the mean DN of the region in question. Some pixels within the region will represent surface elements tilted toward the Sun by a given angle and will have DNs higher than that of a flat element (see Figure 5), whereas on average a similar proportion will represent elements tilted away from the Sun by the same angle and so will have lower DNs. Most of the curves shown in both Figures 5a and 5b are close to linear for tilts within a few degrees of zero, implying that on average, the brightness increases will almost exactly compensate the decreases and that the mean of all the DNs will be a good approximation to the DN of an untilted element. This approximation should hold quite well for most geographic regions at the ~1.1-km nadir resolution of AVHRR images. Mean DNs of 48 and 72 were adopted as the values of DN_0, equal to $DN(i, e, t = 0)$, to be used for a flat element of the ground surface to the north and south of the plume, respectively. However, the approximation is not likely to hold very well for the pixels representing the plume, for two reasons. First, examination of Figure 1a implies that there may not be the same number of pixels tilting toward and away from the Sun (imagine the azimuthal Sun angle is along the axis of the plume). Second, we expect the plume surface to have many undulations (i.e., to exhibit a very wide range of tilts). Thus even if there are an equal number of plume elements facing toward and away from the Sun with the same tilt, the nonlinearity of the curves shown in Figure 5 becomes very important. The sense of the curvature is such that the mean of the plume DNs may significantly underestimate the DN_0 of a horizontal plume surface element. Nevertheless, we initially assume $DN_0 = 77$ for the plume material, found by averaging the DNs for all pixels within the plume boundary.

Brightness ratios are found for each pixel in a column by inserting the appropriate values of DN_0 into equation (6), and B_r values are subsequently converted into tilts using (7). An

Figure 6. Three topographic profiles, as a function of image row number, for column 670 of the Redoubt AVHRR image. The three profiles use DN_0 values of 77, 102, and 127. The scale for the profile is along the left axis. The profiles run from north of the shadow (row 460) to just south of the plume edge (row 570). Also shown are the DN values from the thermal band 4 (scale is on the right axis) for the same column of data.

incidence angle $i = 86.98°$ can be derived from ephemeris information, and an emergence angle $e = 44.65°$ can be determined based on the distance of the plume from the nadir column in the image and the orbit elevation. Because the plume covers only a few tens of kilometers, these values are appropriate to the entire region being analyzed. Topographic profiles are then produced for each column using (8) and (9), where the nominal along-scan pixel resolution ΔX is taken as 1.1 km. It should be noted that for plumes illuminated from the east or west, the nominal pixel resolution along the scan will be a function of the off-nadir viewing angle. The running sum of elevation changes starts from a zero height level defined to be the foot of the shadow. Figure 6 shows an example of one such profile, for column 670 of the image. The curve labeled 77 has assumed $DN_0 = 77$ as discussed above. Also shown in this figure are two other profiles calculated using $DN_0 = 102$ and $DN_0 = 127$ (labeled 102 and 127, respectively). These curves illustrate the consequences of underestimating DN_0. It can be seen from Figure 6 that a 25% variation in the value chosen for DN_0 leads to an error of ~10 m pixel^{-1}. The cumulative effect of this error across the length of a profile for this particular plume is 300 m (10% of maximum plume height).

6. Sources of Systematic Error

There are several sources of systematic error in the model described above. These include the finite resolution of the image data, the choice of DN_0 for a flat plume element, the choice of the multiscattering correction, pixel-to-pixel variations in the emergence angle, and the changing transparency of the plume. The first two of these appear to have the strongest influence on predicted height estimates, and in this section we attempt to assess the magnitude of their impact.

Before analyzing the portion of the profile in Figure 6 representing the plume, recall that the transect begins north of the shadow and continues beyond the southern edge of the plume. Note that the gentle slope in the shadow region (between approximately rows 475 and 510) is indicative of a rise equal to $(90° - i)$ over the length of each pixel. The total rise over the length of the shadow is analogous to the plume altitude derived by the shadow method and provides an absolute altitude for the northernmost plume pixel in the transect. Note also that at the southern plume edge the elevation does not return to an absolute level comparable to that of the shadow foot. This is simply the consequence of the fact that the photoclinometry algorithm deals only with relative heights as implied by pixel-to-pixel brightness changes. The algorithm is incapable of detecting, let alone making any allowance for, the fact that there is a discontinuity in absolute elevation between the last pixel representing the plume edge and the first pixel representing the ground far below it.

The northern parts of all three profiles in Figure 6 are plausible representations of the top of an eruption plume casting a shadow. However, there is no strong indication of the expected steeply sloping southern edge of the plume. To understand this effect, we look again at Figure 5. The curves shown in Figure 5, relating changes in the brightness ratio to the tilt of the scattering surface, are calculated on the assumption that the detector measuring the reflectances has essentially infinite spatial resolution. This would imply that radiant flux can be measured from any very steeply sloping feature (such as the edge of an eruption plume) even though that feature may be very foreshortened by the viewing perspective. It is easy to imagine, however, that a plume edge rapidly sloping down would only occupy a very small fraction of an AVHRR pixel. The finite resolution of satellite images (~1.1 km at best for the AVHRR) ensures that single pixels situated at the southern edge of the plume shown in Figure 2 are likely to contain the flux from plume surface elements that have a much wider range of tilts than pixels located near the top of the plume. Furthermore, some pixels will receive flux from both the plume edge and the underlying surface. Both these factors involve the mixing of high flux values from the nearly vertical edge of the plume with low flux values from either the ground or the less steeply sloping parts of the plume top. This leads to an underestimate of the flux and hence an underestimate of the steepness of the plume at locations near the plume edge.

To explore the importance of this effect, we used equation (7) to synthesize the reflectances of a plume with a hemispherical top and computed the DNs that would be measured if such a plume were imaged by a detector system at various spatial resolutions. We then used these DNs to generate a topographic profile at each resolution. Figure 7 shows the results in a series of cases where the numbers of image pixels occupied by the plume were chosen to be 80, 40, 20, 10, and 5. The

Figure 7. Sensitivity of topographic profiles to image spatial resolution. Each profile refers to the same synthetic hemispheric plume, illuminated from the right with its shadow cast to the left. The x and y axes indicate the horizontal and vertical distances relative to the highest point in the profile. The number labeling each curve is the number of image pixels occupied by the plume. The vertical and horizontal scales are the same in this diagram, emphasizing the fact that profiles composed of less than several tens of pixels cannot faithfully reproduce the shape of the sunward facing edge of the plume. However, all of the profiles with more than 10 pixels locate the plume top quite accurately.

Figure 8. Bulk temperature of a volcanic eruption plume rising in the U.S. Standard Atmosphere. The solid line represents the plume temperature as a function of altitude and the dashed line indicates the tropopause. Note that the plume continues to cool above the tropopause (~11 km) due to adiabatic expansion, even though the entrained air is at a constant temperature.

20-pixel case most closely approximates the resolution of Figure 2, and clearly it is pointless to expect either the shape or the position of the boundary of the sunward facing edge of a plume of this size to be accurately represented in any analysis of an image with this resolution (i.e., any resolution lower than ~1 km $pixel^{-1}$). However, the height of the top of the plume relative to the root of the shadow it casts is quite well represented even in the 10-pixel case. This implies a high confidence in estimates of the surface topography for plume tops (away from the edges) that are at least 10 pixels across in the direction of the transect.

The aforementioned image resolution issues aside, we must still deal with the fact that however plausible the profiles in Figure 6 may appear, we have no way of knowing which of them gives the best estimate of the plume top topography unless we can find an independent method of estimating the value of DN_0 for the plume material. Such a method, relying on plume temperatures, may exist.

In addition to the profiles calculated from the reflectance data in the visible channel, Figure 6 also shows the DNs recorded in AVHRR band 4, one of the two thermal infrared channels. These DNs correspond, more or less linearly, to the temperature of the surfaces represented by the pixels. Note that the minimum DN value in band 4 (coldest temperature recorded) corresponds to the same pixel as the highest point of the plume for $DN_0 = 127$. All of the plume models mentioned in the introduction predict that a volcanic eruption column will have its minimum temperature at its highest point. This statement holds even for columns that penetrate well into the stratosphere. To illustrate this point, Figure 8 shows the bulk temperature of a volcanic plume rising in the U.S. Standard Atmosphere (using the *Glaze et al*. [1997] model). The tropopause for this atmosphere lies at about 11.5 km above sea level, and the plume rises to a final height near 22 km. Despite a constant temperature or even warming of the ambient air with altitude, the plume continues to cool as a result of adiabatic expansion of the gases, attaining its minimum temperature at its maximum height. This minimum plume temperature can be significantly cooler than the air temperature at the same altitude [*Woods and Self*, 1992]. While some care should be taken when calculating profiles for plume material that has moved away from the vent (warming as it equilibrates with its surroundings), in general this method for locating the maximum plume height will work adequately in the area of interest.

On the basis of the foregoing discussion, the pixel containing the minimum DN in the thermal channel for each column is assumed to be the location of the physical top of the plume for that column. By definition, the pixel corresponding to the top of the plume must be the closest approximation to the reflectance of a locally horizontal element of plume material. Therefore we use the band 1 (visual) DN for that pixel as the estimate of DN_0 for all those pixels in the column that had been identified as being part of the plume. Figure 9a shows the topographic profiles for every tenth column of the image between columns 560 and 700, inclusive. For clarity of presentation, the profiles are offset vertically from one another by 500 m.

The majority of these 15 profiles show the kind of topography expected from visual examination of the visible wavelength image. A few (columns 560, 580, 690, and 700), however, behave in an unexpected manner on account of the above algorithm's having selected an inappropriate pixel as the location of the plume top. This error does not appear to be due to the presence of noise in the temperature values: close examination of the thermal DNs (AVHRR band 4) shows a smooth trend in temperature in the case of all of the columns analyzed. Rather, the mismatch between lowest temperature and plume top position may be due to the presence of large-scale convection cells within the plume driven by residual vorticity from the

Figure 9. Topographic profiles (as a function of image row number) for every 10th column of the region identified in Figure 2. (a) Values for DN_0 (flat plume elements) have been found by assuming that the top of the plume occurs at the location of minimum thermal DN. (b) Flat plume elements for columns 560, 580, 690, and 700 have been interpolated from adjacent profiles. In both Figures 9a and 9b the successive profiles, labeled by their column numbers, are offset vertically by 500 m for clarity. The foot of the shadow cast on the ground by the plume is indicated by an asterisk in each case.

Figure 10. (a) Topographic surface of the Redoubt plume as constructed from successive profiles, such as those shown in Figure 9b. The plume shadow appears as the gentle slope to the north of the plume. The profiles are artificially brought back down to a reference altitude of zero beyond the southern edge of the plume. (b) The variation of plume top height as a function of column number in the downwind direction. The top (solid) curve shows the heights found from the full photoclinometric analysis; the bottom (dashed) curve shows the result of using only the length of the shadow to deduce the plume height, thus systematically underestimating the height.

vertical eruption column. Figure 9b shows the result of discarding the original choice of plume top position in the case of these aberrant profiles and using instead the location found by interpolating between the plume top positions on the two adjacent profiles.

7. Discussion

Figure 10a shows a topographic surface for the Redoubt plume that has been built up from successive profiles. Note that the steady slope to the north of the plume is the shadow region, and that the absolute elevation has been artificially brought back down to zero beyond the southern edge of the plume. It can be seen from Figure 10a that the height of the plume generally increases downwind from the vent. Figure 10b shows this height variation (top, solid curve) for the highest point in each profile. Also shown (bottom, dashed curve) is the height that would have been inferred if it had been assumed (as has commonly been done) that the plume top was at the same height as the root of the shadow cast by the plume (see Figure 3).

According to Figure 10b, the shadow length method would systematically underestimate the plume height, in this case by ~30%. Furthermore, the image analyzed has a particularly high incidence angle (~87°); at lower incidence angles the discrepancy would be significantly greater (Figure 3). To gauge the importance of this, it may be recalled that maximum plume height can be used as an indicator of mass eruption rate of magma from the vent. The mass flux is directly proportional to approximately the fourth power of the plume height for maintained plumes from relatively small vents [*Wilson and Walker*, 1987]. Thus, an underestimate of 30% in the plume height leads to an underestimate by a factor of 3 in the implied eruption rate. This would in turn lead to major errors in any predictions made from an analysis of this type of time-dependent processes such as the accumulation rate on the ground of ash falling from the plume.

In the image we have analyzed here, the azimuth of the Sun was nearly perpendicular to the azimuth of the downwind axis of the plume. This orientation is optimal for producing multiple topographic sections through the plume's upper surface (Figure 10a). The dependence of the reflectance function (equation (1)) on the component of tilt of any surface in the plane normal to the plane of solar incidence is very small [*Efford*, 1991]. As a result, images in which the solar incidence plane lies close to the downwind axis of the plume will yield useful information about the plume top height variations along the plume axis, but essentially no information about the plume shape perpendicular to the axis. As a note, the more general rise angle can be accommodated by the angle ϕ in Figure 4b. One can easily derive a value for ϕ from the data in Figures 10a or 10b. The geometric equation with the inclusion of this cross-scan tilt is similar to equation (3) but significantly more complicated and will be presented in the future.

The issue of plume opacity is central to accurate determination of plume top topography. The pixel-to-pixel variations in the scattering properties of the upper surface of the plume are affected by horizontal spreading and diffusion as well as particle sedimentation. To investigate the validity of the assumption that the scattering variations are minimal, we have examined the values chosen for DN_0 in each column. Figure 11 shows the variation in DN_0 values for plume material as a function of distance across the image in the downwind direction. For a uniform surface, we would expect to see similar DN values for all the "flat" pixels. However, what we see in Figure 11 is a general increase followed by a decrease. We interpret the initial brightening of the plume with distance from the vent as representing the condensation of magmatic and entrained atmospheric water vapor as the plume rises. The increasing altitude of the plume over this distance is clearly reflected in the plume's shadow (Figure 2). Due to the lighting geometry of this particular plume, we are only calculating brightness ratios along individual columns; thus pixel-to-pixel variations in w between columns are not significant. However, the plume appears to level off at around column 660 (Figure 10b). The decrease in DN_0 values also occurs at this same point. We interpret this decrease as the region in which the plume is becoming progressively more transparent as material is sedimented out of the plume and the remaining material diffuses in the atmosphere. As the plume becomes more transparent, the darker underlying ground material affects the radiance received at the detector. This is indicated by visual examination of Figure 2, which shows spatial variations of brightness characteristic of albedo variations on the underlying ground in the distal parts of the plume. As was mentioned previously, the photoclinometry technique is only valid where the plume is opaque. Therefore, on the basis of the indication in Figure 11 that the plume becomes transparent around profile 660, the surface shown in Figure 10a has been cut off at that point where the data become unreliable. A more detailed study of the model's sensitivity to specific criteria for defining plume "transparency" will be investigated in future work.

Noting that the spatial resolution of Figure 2 in the east-west direction is approximately 1.1 km pixel^{-1}, Figures 10a and 10b show that on December 16, 1989, the Redoubt plume rose to a height of 3 km while traveling a distance of ~150 km from the vent and subsequently reached a height of ~3.3 km by the time it had traveled 200 km. It is important to remember that these heights relate to the properties of the plume over a range of times from current activity (i.e., over the vent) to at least several hours before the AVHRR scene was obtained (i.e., the downwind extent of the plume). Thus we may be able to use spatial variations in plume height to infer temporal variations in the eruption rate. A sudden vigorous pulse in the eruption rate should be preserved in the altitude of the downwind component of the plume. Using the plume dynamics modeling results of *Wilson and Walker* [1987], a rise height of 3.3 km corresponds to a magma release rate at the vent of 3.8×10^4 kg s^{-1}. The fact that our observations from the AVHRR scene show that much of the plume was at an almost constant altitude implies that over the time period of its generation the eruption rate remained fairly constant. Indeed, combined with ancillary wind velocity information, this may be an effective method for remotely assessing temporal changes in the level of activity during explosive eruptions.

Currently, the nonexistence of comparable topographic data makes it very difficult to validate the application of this technique to volcanic plumes. However, future spaceborne sensors may be able to provide laser altimetry data or stereo images of eruption plumes that can be used for this purpose. Both the multiangle imaging spectroradiometer (MISR) and the advanced spaceborne thermal emission and reflectance radiometer (ASTER) are expected to launch in 1999 as part of the first Earth Observing System (EOS) platform and, unlike the SPOT sensors, will view the Earth's surface at multiple angles in the along-track direction [*Diner et al*., 1989; *Kahle et al*., 1991]. Thus a plume could be imaged from several angles within a few minutes, thereby creating good stereo pairs. However, problems may exist in the automatic selection of camera pairs on MISR (which views the Earth at nine different angles from ±72.5° from nadir) because of the extreme parallax that exists. In many cases it may be difficult to identify the same part of a plume if the look angle differences are more than ~30°, resulting in a major limitation on obtaining successful stereo solutions. A second aspect of the MISR instrument is that particularly at the extreme off-nadir angles (±60° and ±72.5°), an eruption plume would be viewed at a sufficiently oblique angle that it would be seen almost in profile. MISR data can be obtained in high-resolution mode, so that they have a spatial resolution of ~700 m. For a plume that rises ~15–18 km above the surface, this might mean that the vertical extent of the plume would comprise 20–25 MISR pixels. ASTER has only one forward-looking camera and a nadir view, so there are no opportunities to optimize the viewing conditions. However, ASTER will also obtain nadir-looking thermal infrared data so that the temperatures of the plume top surface may be correlated with any stereo model.

Figure 11. Variation of DN_0 for the plume (proportional to the geometric albedo) as a function of column number.

While we hope that one of these future instruments will have the opportunity to view an active volcanic plume, providing an independent method for estimating plume topography, it should be noted that ASTER and MISR have repeat viewing cycles of several days. The primary implication of the long repeat cycle is that the probability of one of these instruments actually acquiring images of a volcanic plume is greatly reduced. However, the acquisition of data by either MISR or ASTER would provide the ideal opportunity to test the photoclinometry technique. Also flying on the same EOS satellite platform will be the Moderate-Resolution Imaging Spectroradiometer, or MODIS. MODIS will have 36 channels, including channels that are very similar to the five AVHRR channels, and will have a spatial resolution comparable to the AVHRR. The technique described here for AVHRR data is directly applicable to MODIS. The plume surface topography generated from the MODIS data could easily be compared to simultaneously acquired stereo modeled topography derived from either MISR or ASTER data. In addition to the AVHRR analog channels, MODIS will also have several channels continuously acquiring data with much higher spatial resolution (250–500 m pixel^{-1}). The edges of plumes observed in these channels will be much easier to resolve.

8. Conclusions

We have shown that photoclinometric analysis can provide a means of estimating heights and three-dimensional topographic surfaces of volcanic plumes from digital images without recourse to stereoscopic techniques. The length of the shadow and the elevation of the Sun provide the means of converting relative heights along a plume's upper surface to absolute heights above the shadow foot. It is also possible to determine aspects of the morphology of the plume top, which may have significant local undulations due to residual vorticity from the buoyant column, gravity current dynamics, or imposed atmospheric turbulence. The technique can be applied to visible satellite images of volcanic plumes that have uniform scattering properties, and is most effective on images with moderate incident Sun angles and plume transport perpendicular to the azimuth of the Sun. The resulting plume height estimates are more accurate than shadow-derived heights and more reliable than temperature-derived heights.

We have investigated two important sources of systematic error: the finite resolution of the image data and the choice of DN value for a flat plume element. The finite resolution of satellite images leads to errors in the estimation of the topography of all parts of the upper surface of an eruption plume, having its greatest effect on those parts of plumes that are inclined most steeply towards the Sun. However, provided that the plume spans at least 10 image pixels, useful results can be obtained for the height of the plume top relative to the root of the shadow. An underestimate in the reflectance of a horizontal plume element can translate into a height error of 10–20 m pixel^{-1} (for a 1.1-km pixel) that accumulates over the length of each profile. Use of the mean of the DNs for the plume studied here leads to an error of at least 10% in the inferred plume top height. Such an error would translate into almost a 50% error in the implied eruption rate from the vent. However, thermal infrared data can be used to locate the pixel in each profile with the lowest temperature. In general (73% of the profiles considered here), the lowest temperature on the upper surface of the plume corresponds to the location of the highest point on the plume profile. Using the visible DN value for the pixel where the minimum temperature occurs as representative of the flat plume surface reflectance leads to much greater confidence in estimated plume heights. Future work will attempt to quantify the model's sensitivity to other sources of systematic error.

While it is desirable to have stereo images of volcanic plumes from the ATSR, MISR, or ASTER instruments, the repeat viewing cycles of these instruments are such that the probability of capturing an active volcanic plume is very low. The technique presented here will allow the determination of surface topography as well as absolute plume height from basic visible satellite images that will be readily available from MODIS as well as the AVHRR instruments. The combined viewing opportunities of MODIS and AVHRR increase the probability of viewing an active volcanic plume tremendously, and so it is most likely that any new eruption plume will be detected in images obtained only in nonstereo mode. Hence the photoclinometric method will provide a fast and quantitative estimate of the initial vigor (i.e., the magma mass eruption rate) of a newly detected eruption, as well as information on the dynamics of the plume emplacement and transport.

Notation

A	azimuthal Sun angle, deg.
B_r	brightness ratio.
b, c	parameters expressing relative amounts of scattering.
DN	digital number.
DN_0	digital number for a flat pixel element.
e	emergence angle for a flat pixel element, deg.
e_t	emergence angle for a pixel tilted through the angle t, deg.
F	energy flux reaching the satellite detector.
H	height of the pixel element.
ΔH	relative height change across ΔX.
i	incidence angle for a flat pixel element.
i_t	incidence angle for a pixel tilted through the angle t.
K	constant accounting for intensity of illumination source.
t	angle of tilt in the x direction, deg.
w	single-scattering albedo.
x	axis direction that is closest to A.
ΔX	nominal pixel resolution.
y	axis direction that is perpendicular to x.
Θ	parameter characterizing the RMS surface roughness.
Φ	function of i, e, Ψ, w, b, c, and Θ
ϕ	angle of tilt in the in the y-direction, deg.
ψ	angle of vision, deg.

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