Antipodal seismic reflections upon shear wave velocity structures within Earth's inner core

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1. Introduction

Earth's core is central to our understanding of many challenging geophysical inquiries. The geodynamo and generation of Earth's magnetic field (Gubbins, 1977; Loper, 1978; Braginsky, 1963) are perhaps the most evident to people. Research suggests that the magnetic inertia of the inner core moderates geomagnetic reversals (Gubbins, 1999; Roberts, 2008), producing a relatively stable external dipole field (Hollerbach and Jones, 1993). The geomagnetic secular variation (Aubert et al., 2013) and the eccentricity of the geomagnetic dipole (Olson and Deguen, 2012) may be linked with differential inner core growth. Illuminated by earthquakes in the crust and upper mantle, and observed by seismic observatories at Earth's surface, seismology offers the only direct means to investigate the inner core and its processes.

The growth of the inner core (Yoshida et al., 1996) within the liquid outer core is a key process affecting the temporal evolution of Earth and its internal composition (Birch, 1952; Jacobs, 1953). The relationships among heat flow at the inner-outer core boundary region (Sumita and Olson, 1999; Verhoogen, 1961; Aubert et al., 2008), melting and freezing of the inner core (Fearn et al., 1981; Loper and Roberts, 1981; Monnereau et al., 2010; Alboussiere et al., 2016; Gubbins et al., 2011; Cormier and Attanayake, 2013; Attanayake et al., 2014; Pejić et al., 2017), and outer core convection (Gubbins et al., 2004) focus on boundary processes. Rotation of the inner core with respect to the mantle (Song and Richards, 1996; Tsuboi and Butler, 2020) has links to outer-core and inner-core viscosities (Buffett, 1997), inner-core topography (Cao et al., 2007), and convective interactions (Lister and Buffett, 1995). Mineral physics (Anderson, 1995) experiments approaching core temperature (Boehler, 1999) and pressure (Lin et al., 2003; Dubrovinsky et al., 2000), ab initio calculation of material properties (Vocadlo, 2007),...
and the constitution (Birch, 1940; Sumita and Olson, 1999) of Fe alloys with lighter elements provide insights gleaned from seismological measurements of the core.

The inner core was discovered by Lehmann (1936) from a compressional wave reflection from its boundary. The solidness of the inner core was first inferred from mineral physics considerations (Birch, 1952), and then from normal mode observations (Derr, 1969; Dziwonski and Gilbert, 1971). Travel time observations through the inner core find cylindrical anisotropy (~1%) with an axis of symmetry aligned with the earth’s rotation axis, faster than equatorial paths (Morelli et al., 1986). Hemispheric compressional velocity heterogeneity is less than 1% between quasi-eastern (40°E–180°E) and quasi-western (180°W–40°E) sectors (Tanaka and Hamaguchi, 1997; Niu and Wen, 2001; Wen and Niu, 2002; Stroujkova and Cormier, 2004; Attanayake et al., 2014; Waszek and Deuss, 2011). These quasi-hemispheric structures continue, and anisotropy increases, through a depth range 150–220 km (Garcia et al., 2006) to 550 km depth (Cormier and Attanayake, 2013). Normal modes are compatible with an isotropic layer of up to 275 km thick at the top of the inner core (Irving and Deuss, 2011). The innermost inner core below 700 km depth (~500 km radius) has been probed by compressional waves, wherein the anisotropy is referenced from the axis of rotation (Ishii and Dziwonski, 2002; Cormier and Stroujkova, 2005; Cao and Romanowicz, 2007). The rotation of the inner core with respect to the mantle has been proffered in many studies (Song and Richards, 1996; Souriau et al., 1997; Tromp, 2001; Tsuboi and Butler, 2020). The primary constraint upon shear wave velocity in the inner core comes from normal mode data (Dziwonski and Gilbert, 1971, 1972; Deuss, 2008; Lythgoe and Deuss, 2015; Robson and Romanowicz, 2019), where the estimated inner core shear velocities are remarkably consistent, and do not deviate significantly from the initial starting model value of $V_S \sim 3.5$ km/s (Dziwonski and Gilbert, 1971, 1972)—the pioneering work of Derr (1969) shows one starting model (HT11GS1) with $V_S \sim 2.5$ km/s converging to $V_S \sim 2.18$ km/s. The observation of a J phase, a shear wave within the inner core, has been proposed by a number of researchers (Julian et al., 1972; Tkalcic and Pham, 2018) near epicentral distances and travel times that are concordant with the normal mode estimate of inner core shear velocity; e.g., Tkalcic and Pham (2018) propose inner core shear velocities within 2.5% (slower) than the Preliminary Reference Earth Model (PREM—Dziwonski and Anderson, 1981).

2. Antipodal data and observations

Complementing the body wave and normal mode research on the core structure and properties as highlighted in the Introduction, antipodal studies provide unique constraints. In propagating through the core to the earthquake’s antipode at $\Delta = 180^\circ$, the focusing of waves near the antipode amplifies seismic energy (Rial and Cormier, 1980; Butler, 1986; Niu and Chen, 2008; Butler and Tsuboi, 2010; Cormier, 2015; Attanayake et al., 2018; Tsuboi and Butler, 2020; Butler and Tsuboi, 2020). Near the antipode (>179°) of an earthquake, the seismic energy from all azimuths about the earthquake source coalesces together, and the individual ray paths merge into a ray surface. The ray surfaces comprising the antipodal propagation circumscribe the Earth about the diameter between earthquake source and receiving antipodal station. Although only PKIKP propagates along this diameter (Fig. 1) and is not antipodally amplified, we find it convenient to review and discuss antipodal data diametrically as designated by their unique diameters. At distances less than antipodal where ray paths (e.g., PKIKP, Fig. 1) may sample a wavelength-size “patch” on the inner core boundary, the antipodal phase PKIKP samples a great circle region encompassing the inner core boundary orthogonal to that diameter. Furthermore, for a PKIKP wavelength of 50 km on the inner core boundary (ICB) the antipodal phase samples about two orders of magnitude more of the ICB

![Fig. 1. Maps show the locations of earthquakes (red symbols) and corresponding antipodal stations (names highlighted in red). Locations of the antipodal seismic stations are labeled (yellow pins). Inset figure shows the antipodal ray paths for PKIKP (cyan), PKIKP (green) and the suggested paths for the central core phases, PKI100,IKP (red) and PKI250,IKP (blue), as observed at TAM and PTGA. Path segments propagating in the Mantle, Outer Core, and Inner Core, are designated P, K, and I, respectively. Whereas PKIKP (green) reflects beneath the inner core boundary, PKI100,IKP (red) reflects beneath an interface in the upper inner core, demarcated by a dotted red circle at 100 km depth. Similar, PKI250,IKP (blue) reflects beneath an interface in the upper inner core, demarcated by a dotted blue circle at 250 km depth. Although ray paths are shown in this cross-section, the antipodal energy converges from all azimuths as a ray surface. The abridged names for PKI100,IKP and PKI250,IKP in the text are PKI100 and PKI250, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image-url)
than the non-antipodal wave.

Seismic data from five diameters are examined (Fig. 1)—Tonga to Algeria, Sulawesi to Amazon, northern Chile to Hainan Island, and two between central Chile and the mainland China. Each of these diameters have been previously examined for PKIKP and PKI potassium (Tsuboi and Butler, 2020; Butler and Tsuboi, 2010), where significant spatial and temporal heterogeneity was observed. This reconnaissance seeks to understand where the significant seismic energy propagating prior to PKIKP arises—between PKIKP and PKI potassium, and after near-source, surface reflections like pPKIKP. Observed both in stacked and individual waveforms, these arrivals may correspond to significant reflectors in the upper central core—the “central core” in this study refers to the inner core below the uppermost inner core and above the innermost inner core, between radii 1121.5 and 500 km. To enhance the signal-to-noise ratio (SNR) of the pre-PKIKP data, we collected available earthquake data antipodal to TAM.G, PTGA.IU, QIZ.CD, ENH.IC, and XAN.IC seismic stations (Fig. 1).

2.1. Data set

All data (BHZ) were downloaded from the IRIS Data Management System. The earthquake events are listed in Supplement Table S1, subdivided by antipodal station. The numbers of station-events are TAM (26), XAN (15), ENH (9), PTGA (4) and QIZ (4). The 20 samples/s, vertical data were converted to displacement (m) and high-pass filtered using a two-pass, elliptical filter (high-pass corner = 1/50 Hz, 3 poles, dBStop = 50, Ripple = 2). Seismic record sections aligned on PKIKP are plotted for TAM (Fig. S1–S3), PTGA (Fig. S4), XAN (Fig. S5), QIZ (Fig. S6), and ENH (Fig. S7), each showing the relative move-out of PKIKP.

Fig. 2 plots selected waveforms from Figs. S1–S7 for each antipodal station, which are used in synthetically modeling features highlighted from stacking the antipodal data (Figs. 3 and 4). The waveform data for TAM and PTGA show significant arrivals (“?” and “??”) marked in red between PKIKP and PKI potassium. These apparent phases arrive ~7 and ~17 s before PKIKP. For the Chinese stations—QIZ, ENH, and XAN—the dichotomy with TAM and PTGA is striking. For ENH the evidence for arrival (“?”) is subdued; for XAN waveform data do not evince the arrival (“??”). QIZ is not well resolved. Given that wave propagation paths for the five diameters do not share common paths in the crust, mantle, or outer core, we employed data stacking for each of the five diameters to improve their signal-to-noise ratios.

2.2. Stacking the data

Data are stacked in several ways to increase SNR. Initially, we simply aligned the signals on PKIKP (normalized to PKIKP ≈ 1 at PKIKP time = 0 s, as shown in the record section plots, Figs. S1–S7) and derived the mean amplitude stack as a function of time. We also employed phase-weighted stacking (Schimmel and Paulssen, 1997)

\[ g(t) = \frac{1}{N} \sum_{k=1}^{N} s_k(t) \left| \sum_{k=1}^{N} e^{i \Phi_k(t)} \right| \]

where \( s_k \) and \( \Phi_k \) are each averaged as the amplitude stack and instantaneous phase stack over the \( N \) traces—the squared phase stack used herein (\( \zeta = 2 \)) can be regarded as the power of the phase stack that helps to increase the SNR (Schimmel and Paulssen, 1997). For \( \zeta = 0 \), the stack reverts to amplitude stacking only. Both methods are shown in Fig. 3. Most of the antipodal events are thrust earthquakes. In stacking both normal and thrust events, the normal events are inverted; i.e., \( \zeta_{\text{normal}} = -1 \). Data are independently stacked for each station, employing various subsets of the data based upon distance from the antipode.

Depth phases potentially interfere with the stacking where arrivals of pPKIKP and sPKIKP overlap with inner core phases (in principle, the \( sP \) reflection at vertical incidence is negligible from a flat, horizontal interface; e.g., Tsuboi and Butler, 2020). The range of \( pP \) and \( sP \) arrival time windows are plotted in colour over the stacked data (Fig. 3), showing good separation from candidate precursory phases. With few exceptions (Table S1), we limit the earthquake depths <50 km.

We do not imply that the precursory arrivals are predetermined as from the inner core. Nonetheless, in reviewing other possible propagation paths for the precursory energy before PKIKP (Butler and Tsuboi,
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A) Amplitude Stack

- PKIKP stack
- PKIKP time
- pPKIKP range
- ± pPKIKP (median)
- sPKIKP range

TAM 179.9° to 179.4°Δ
TAM 179.3° to 179.0°Δ
TAM 178.9° to 178.3°Δ

Time with respect to PKIKP, sec

B) Phase Stack

- PKIKP stack
- PKIKP time
- pPKIKP range
- ± pPKIKP (median)
- sPKIKP range

TAM 179.9° to 179.4°Δ
TAM 179.3° to 179.0°Δ
TAM 178.9° to 178.3°Δ

Time with respect to PKIKP, sec

C) Amplitude relative to PKIKP

- PKIKP stack
- PKIKP time
- pPKIKP range
- ± pPKIKP (median)
- sPKIKP range

PTGA 179.9° to 179.5°
QIZ 179.6° to 179.3°
ENH 179.3° to 179.0°
XAN 179.4° to 179.0°
XAN 178.9° to 178.2°

Time with respect to PKIKP, sec

D) Amplitude relative to PKIKP

- PKIKP stack
- PKIKP time
- pPKIKP range
- ± pPKIKP (median)
- sPKIKP range

PTGA 179.9° to 179.5°
QIZ 179.6° to 179.3°
ENH 179.3° to 179.0°
XAN 179.4° to 179.0°
XAN 178.9° to 178.2°

Time with respect to PKIKP, sec

(caption on next page)
Fig. 3. Amplitude (AC left) and phase (BD right) stacks of antipodal data are plotted for TAM and CD) PTGA, QIZ, ENH, and XAN. Stacked waveforms are aligned with PKIKP time and amplitude. Three near-antipodal distance ranges are plotted for TAM. Near-antipodal distance ranges are plotted for PTGA, QIZ, ENH, and two ranges are shown for XAN. Numbers of events associated with the stacks are indicated at right. The arrival time windows of pPKIKP (red) and sPKIKP (white dotted) are indicated; the green zone is the median pPKIKP time ± median absolute difference, indicating the window where 50% of the pPKIKP arrivals occur. The arrival time window of pPKIKP is indicated in blue. Following PKIKP and before the arrival of PKIIKP are two precursory phases tentatively labeled PK1-JKP and PK1+JKP. The former is aligned (violet vertical line) and marked with a violet star; the latter is indicated (dot-dash violet line), preceding PK1-JKP; both extend beyond TAM and PTGA plots. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 4. Phase-weighted slant stacks are shown for the two antipodal diameters (TAM and XAN) with the largest number of events contributing to the stacks. For TAM we show two range of distances in the stack. The ranges of ray parameters between $p = 0$ (green) and $p = 2$ (red) is plotted in 0.2 s/° increments in black. The PKIKP arrival window is highlighted in blue. Slant stacking PKIKP for $p = 1.8$ s/° amplifies the arrival coincident with the expected slowness for PREM. For the precursor to PKIKP designated as PK1-JKP, the slant stacking indicates that the maximum amplitude occurs near the ray parameter for PKIKP. Similarly, the earlier PKIKP precursor is amplified near the ray parameter for a phase designated PK1-JKP. Note that the slant stack for XAN at PKIKP (i.e., 4C) is influenced by near-source surface reflections for $p > 0$ s/° (Fig. 4C). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

2010; Tsuboi and Butler, 2020; Niu and Chen, 2008), P-wave scattering contributions within the uppermost inner core above ~250 km depth are limited by wavelength (~55 km) of PKIKP and PKIIKP. Array measurements of 1 Hz PKIKP coda (Vidale and Earle, 2000) may be modeled as 1.2% variation in elastic moduli and density with a correlation length contributions within the uppermost inner core above ~250 km depth. Cormier (2007) modeled vertical and horizontal texture measurements of 1 Hz PKIKP coda (Vidale and Earle, 2000) may be modeled beyond our resolution window. Cormier (2007) modeled vertical and horizontal texture variations in the fabric of the inner core to understand the 1 Hz coda of PKIKP and PKIIKP. Scattering contribution from elsewhere (e.g., Wang and Song, 2019) in the mantle and core lacks antipodal focusing and is subject to geometric spreading effects when the ray parameter is re-set upon scattering incidence (Ishimaru, 1978; Tsuboi and Butler, 2020).

The amplitude and phase-weighted stacks in Fig. 3 show a rough demarcation between stations where seismic arrivals between PKIKP and PKIIKP are large (TAM and PTGA) and small/ negligible (XAN, ENH, and QIZ) excepting only the near-source, depth phase pPKIKP. The latter stations each located in China, antipodal to Chile, have an aperture of $\Delta = 15^\circ$, whereas TAM and PTGA are separated by $\Delta = 67^\circ$. In stacking the data in Fig. 3 we confirm our initial assessment that the energy between pPKIKP and PKIKP is coherent both on the amplitude and phase-weighted stacks.

The timing (~7 and ~17 s) of these stacked precursors to PKIKP corroborates the waveform timing in Fig. 2. Furthermore, the stacked data authenticate findings in Fig. 2 that there are two distinct groups — TAM and PTGA with clear PKIKP precursors and QIZ, ENH, and XAN where evidence is subdued. Hypothesizing that the energy between PKIKP and PKIIKP propagates deeper and earlier through the inner core than PKIKP (which is a maximum time phase), the precursor phases are tentatively designated PK1-JKP and PK1+JKP, reflecting their uncertain origin in the inner core. Fig. 3 clearly shows the parallel (in time) arrivals observed at TAM and PTGA for both PK1-JKP and PK1+JKP. The TAM stacks are also plotted in three antipodal-distance ranges, wherein amplitudes decline for distances $\Delta < 179^\circ$.

We also employed slant-stacking (Niu and Chen, 2008), shifting the seismogram by $r_m = p(180^\circ - \Delta_m)$ seconds for the nth station over a range of ray parameters ($p = 0$ to 2 s/°)—note that the prior methods are
simply slant stacks with \( p = 0 \). We show in Fig. 4 the slant-stacks for TAM and XAN, incorporating data from \( \Delta = 179.9^\circ-178.2^\circ \). Slant stacking the XAN data band overwhelmingly increases the amplitude (by a factor of 3) of the \( PKI\nabla IKP \) precursor labeled \( PKI\nabla IKP \), which appeared negligible in Figs. 2 and 3. Slant stacking the 9 events for ENH was inconclusive over the narrow range of distances available (179.1–179.2°). We observe in Fig. 4 that in slant stacking between ray parameters for \( PK\nabla IKP \) (\( p = 0 \) s/°) and \( PKI\nabla IKP \) (\( p = 1.8 \) s/°) shows significant amplitude increases for the phases labeled \( PKI\nabla IKP \) and \( PKI\nabla IKP \) for ray parameters approaching that of \( PKI\nabla IKP \). This observation strongly supports the hypothesis that the precursor phases are related to \( PKI\nabla IKP \). Furthermore, the energetic precursor \( PKI\nabla IKP \) exceeds the amplitude of \( PKI\nabla IKP \).

Looking forward, the study of Huang et al. (2015) shows the exceptional possibilities for global array processing of \( PKI\nabla IKP \) and \( PKI\nabla IKP \). Using the USAarray as a continent-wide, broadband seismic interferometer, the global earthquakes \( MW > 7.0 \) were stacked for the phases that propagate through the core, reflect at the Earth’s opposite surface, and then return through the core: \( PKI\nabla IKP \) (\( PKI\nabla IKP \)) and \( PKI\nabla IKP \). Although not antipodal (the closest earthquake is ~35° to the middle of the USAarray), the concept of an array antipodal to a major earthquake zone is enticing. Obviously from this study, an array at Tamanrasset (TAM), Algeria would be valuable scientifically, and provide new tools to unravel the antipodal propagation.

We have used the data stacking to illuminate the path forward in understanding the timing and amplitude of the \( PKI\nabla IKP \) precursors. Nonetheless, we model these observations using the actual antipodal data for earthquakes recorded antipodally at TAM, PTGA, QIZ, ENH, and XAN to understand the amplitude, time, and phase information in the waveforms and synthetic seismograms.

3. Simulations: 3D spectral element modeling

In modeling the antipodal data set, we approached the problem synthetically using the 3D spectral element method (3DSEM—Komatitsch and Vilotte, 1998; Komatitsch et al., 2002; Tsuboi et al., 2003; Komatitsch et al., 2005) as previously applied (Butler and Tsuboi, 2010; Tsuboi and Butler, 2020; Butler and Tsuboi, 2020). The initial model used incorporates a simple PREM mode for the Core, a 3D tomographic model—\( s362\)wmani—for Earth’s Mantle (Kustowski et al., 2008), crustal model CRUST2.0 (Bassin et al., 2000), and ellipticity. In synthesizing the antipodal observations, global CMT mechanisms and locations (Ekström et al., 2012) are used in modeling the earthquake sources. Incorporating a 3D mantle and crust, we include within the 3DSEM synthetics the energy scattered from above the core (e.g., upper mantle, \( D^\circ \) and the core-mantle boundary). Since the 3DSEM synthetics are correct for periods \( T > 3.5 \) s, we employed a two-pass elliptical filter (low-pass corner \( \approx 0.5 \) Hz, 2 poles, dBstop = 100, ripple = \( \frac{1}{2} \)) identically on the data and the 3DSEM synthetic to permit direct comparison.

3.1. Models tested

Although it may be feasible to construct a model where \( PKI\nabla IKP \) arrives 25 s after \( PKI\nabla IKP \) by diffracting or refracting around deep central core structure, achieving a substantial \( PKI\nabla IKP \) amplitude relative to \( PKI\nabla IKP \) is difficult. Furthermore, three studies of \( PKI\nabla IKP \) propagating through the innermost inner core (radius < 500 km) offer no evidence of such structure (Ishii and Dziewonski, 2002; Cormier and Stroujkova, 2005; Cao and Romanowicz, 2007). Waveform studies of the upper inner core show little evidence for compressional wave discontinuities near 100 km depth in the propagation bands 0.1–3 Hz (Stroujkova and Cormier, 2004) and ~1 Hz (Leyron et al., 2005; Yu and Wen, 2006; Cormier et al., 2011). In a data set with a dominant frequency near 0.5 Hz, Cormier and Attanayake (2013) have found a 1% \( V_p \) discontinuity in one 45° spherical lune 140 km below the inner core boundary (ICB) centered beneath the Atlantic Ocean—yet not evident in the seven other spherical lunes. Song and Helmbargar (1998) saw evidence for a \( V_p \) velocity jump of 3.5–5% at 200–300 km below the inner core boundary, reproduced in the broadband (\( T \approx 10 \) s) \( PKI\nabla IKP \) waveforms of the polar paths reasonably well. However, the near equatorial paths, such as sampled herein, fit a smooth PREM-like model.

If \( PKI\nabla IKP \) is reflected by structure below the inner core boundary, by symmetry each of the two path segments contributes ~3.5 s to the 7 s time advance with respect to \( PKI\nabla IKP \). The turning point of \( PKI\nabla IKP \) occurs at ~87 km depth. Hence, the turning point of the \( PKI\nabla IKP \) ray surface is earlier than and below the \( PKI\nabla IKP \) ray surface.

We approached the study of the \( PKI\nabla IKP \) precursors from the perspective of the compressional velocity \( V_p \), casting a wide net of possible \( V_p \) models to match the features of the precursors—principally relative time and amplitude—with respect to \( PKI\nabla IKP \) and \( PKI\nabla IKP \). Fig. S8 plots initial models considered. The PREM model includes a 3D mantle (Kustowski et al., 2008); for model \( V_p \) putting all the change in \( V_p \), a +9% increase at \( r = 1100 \) km (11–12 km/s) is insufficient in matching the \( PKI\nabla IKP \) amplitude but fits the relative timing; \( V_p \) has a low-velocity layer; \( V_p3 \) modifies \( V_p1 \) with a ‘mushy’ \( V_s \) layer above \( r = 1100 \) km; \( V_p4 \) has \( V_p \) unchanged from PREM with \( V_p = 0 \) above \( r = 1100 \) km; and \( V_p5 \) modifies \( V_p4 \) with \( V_p = 0 \) above \( r = 1200 \) km. Of these models, only \( V_p4 \) showed promise with correct timing and phase, albeit the amplitude is too small.

Fig. S9 continues the model series from \( V_p4 \): \( V_p4.2 \) has a \( V_s \) gradient (0–3.5 km/s) from \( r = 1100 \) to 1200; \( V_p4.3 \) keeps \( V_p = 0 \) above \( r = 1100 \) and extends the PREM outer core \( V_p \) into the inner core to \( r = 1100 \) km; \( V_p4.4 \) has \( V_p \) unchanged from PREM and \( V_p \) has a ‘mushy’ gradient (0–0.3 km/s) between \( r = 1221.5 \) km to 1100 km; \( V_p4.5 \) modifies \( V_p4 \) with gradients where \( V_p \) and \( V_p1 \) decrease from \( r = 1221.5 \) to 1100 km; \( V_p4.9 \) modifies \( V_p4 \) with \( V_p = 0 \) above 1100 km and modifies \( V_p5 \) and density \( \rho \) to have no impedance contrast at the IOCB. This series of models with a solid–liquid boundary at 1100 km meets the timing of \( PKI\nabla IKP \), but not the amplitude.

Models were limited to isotropic changes to inner core radial velocities in this paper. Whereas most details of the shear wave structure are unresolved, complexities such as 3D structures and anisotropy clearly are expressed in prior P-wave studies (see Introduction). Although anisotropy may be included in the 3DSEM synthetics, we have not expanded the model space beyond the extensive, isotropic models already considered.

The pattern of cylindrical anisotropy observed over the whole inner core (faster parallel to Earth’s rotational axis; e.g., Morelli et al., 1986) is not clearly resolved in the upper ~200 km of the inner core (e.g., Niu and Wen, 2001; Irving and Deuss, 2011). Further, the antipodal, inner core paths discussed herein differ less than 35° from the equatorial plane. Helfrich and Mainprice (2019) have shown that a simple form of anisotropy—Vertical oriented Transverse Isotropy (VTI)—can enhance a \( PKI\nabla IKP \) reflection signal incident at the top of ICB, without affecting the P travel time. Nonetheless, an underside reflection (e.g., \( PKI\nabla IKP \)) is not similarly affected.

The shear velocity at the inner-outter core boundary (IOCB) affects the reflection coefficient of \( PKI\nabla IKP \) (Cormier, 2015). Following this lead, we examined structures wherein the compressional velocity and density within the inner core are unchanged (PREM), but the shear velocity is varied across a discontinuity at the top of the central core (100 km below ICB). Having tested a plethora of \( V_s \) models, we considered the effect of \( V_s \) on \( PKI\nabla IKP \)—an apparent discontinuity near \( r = 1100 \) km had yielded a clear arrival time, but an insufficient \( P \) amplitude. Modification of \( V_s \) from ~3.5 km/s (PREM) in the inner core was not undertaken lightly, given the weight of normal mode estimates since the 1970s (summarized in Supplement Table S2). Nonetheless, changing the value of \( V_s \) below the solid-liquid interface directly changed the amplitude of \( PKI\nabla IKP \) without altering its travel time relative to \( PKI\nabla IKP \) and \( PKI\nabla IKP \).

3.1.1. TAM and PTGA

The TAM–Tonga axis has the most events and is subdivided by
natural distances ranges (Table S2) in the data to distinguish antipodal focusing (Fig. 3A). The TAM stack for \( \Delta = 179.9-179.4^\circ \) displays the key features of the central core at a glance. Both the phase and amplitude stacks give congruent results, corroborated by slant stack results (Fig. 4). PKIKP exceeds all other arrivals. Given the depth of the earthquakes, the near-source, surface reflected phases (principally pPKIKP) arrive within about 12 s of PKIKP. The expected arrival window of PKIKP (Fig. 1) near 32 s for PREM is highlighted in blue on the traces in Fig. 3. However, an earlier phase near 25 s (violet star) is significant with an amplitude >50% of PKIKP. This arrival termed PKI-IP has heretofore not been observed. Following PKIKP are arrivals (Rial and Cournède, 1980; Butler and Tsuboi, 2010) associated with the whispering gallery of phases (PKIIKP+ PKIIKP + ...) propagating beneath the inner core boundary, and with PKPc.diff which diffracts around the inner-outer core boundary. The TAM stack for \( \Delta = 179.3-179.0^\circ \) is similar to the closest antipodal stack. However, at distances \( \Delta < 179^\circ \), the antipodal focusing at TAM is no longer constructive interference in the phase stack, and only a small arrival is evinced. Nonetheless, the slant stacked data clearly shows PKI-IP in stacked data for \( \Delta > 178.2^\circ \) (Fig. 4).

Two earthquakes antipodal to TAM are modeled in Fig. 5— a normal faulting event in 2011 and a thrust earthquake source from 1992 (Table S1). The latter earthquake is one of three nearly identical triplets previously studied (Butler and Tsuboi, 2010; Tsuboi and Butler, 2020). In these models \( V_S \) and density are unchanged from PREM values in the inner core, and \( V_S \) is modified in steps between 2.5 and 6.0 km/s. Decreasing \( V_S \) below that for PREM decreases the amplitude of PKI-IP, whereas increasing \( V_S \) effectively increased its amplitude. To attain the relative amplitudes observed for PKI-IP/PKIKP, we found that \( V_S \) must be \( \geq 5 \) km/s—Fig. S10 shows this amplitude–\( V_S \) relationship. Fine-tuning the depth of the apparent discontinuity converged to \( z = 100 \) km with a trade-off of \( \pm 2 \) km in depth corresponding with 0.1 s in time. Having a candidate phase that has an appropriate amplitude and arrival time, we recognize that PKI-IP has a standard seismic phase name of \( PKI_{100} \) (Schweitzer et al., 2019) as an underside reflection from an interface at \( z = 100 \) km deep below the ICB.

The PTGA stacked observation of \( PKI_{100} \) (Fig. 3B) is most similar to TAM, both in the stacked amplitude and phase-weighted ratios (>50% of PKIKP), and relative timing. The PTGA stacked data corroborate the TAM observations (Fig. 3). The PTGA dataset encompasses a wider range of earthquake magnitudes, including Mw 7.0 and 7.9 (Table S1). The TAM and PTGA stacks for antipodal distance \( \Delta > 179.4^\circ \) in Fig. 3 show both PKI-IP (PKI_{100}-IPK) and PKI_{100}-IKP. TAM and PTGA do not share common near-source, near-receiver, mantle, or outer-core propagation paths, and therefore the commonalities seen in the TAM and PTGA stacks then resides in shared inner core structure. However, this shared structure must thereby exist over two substantially different sections of the inner core. Given the corroborration of TAM and PTGA observations of PKI_{100}-IPK, we have proceeded within the framework applied to PKI_{100}-IPK, but deeper within the central core.

To model the observation of PKI_{100}-IPK at PTGA we synthesize two models displayed in Fig. 6 (Vs3 and Vs4) to test for a deeper, shear wave interface below PKI_{100}-IPK. Fig. 7 shows that PKI_{100}-IPK is not well fit by PREM (Fig. S8). By timing considerations, an apparent discontinuity near a depth of 250 km (150 km thickness, with a \( V_S \geq 5 \) km/s) overlying PREM structure presents a reasonable fit to the PTGA data. In Fig. 7, the two end-member models considered (Vs3 and Vs4) differ only above \( z = 100 \) km. We see PKI_{250}-IPK in Fig. 7 expressed nearly identically—e.g., the structure between 50 and 250 km does not contribute substantially to PKI_{250}-IPK. However, note that for a \( P \) wavelength of \( \sim 50 \) km, we cannot easily distinguish a sharp interface—whether at 100 or 250 km depth—from a narrow transition gradient. These models are not unique but do capture the essence of the data.

3.1.2. QIZ, ENH, and XAN
The antipodal stacks for QIZ, ENH, and XAN in China and PTGA in Brazil (Figure 3D) show the substantial diversity of the observations of PKI_{100}-IPK for paths between South America and Southeast Asia (Fig. 1). As seen in Fig. 3B, the PTGA stack differs significantly from those for QIZ, ENH, and XAN. For QIZ and ENH, there is no firm evidence of a PKI_{100}-IPK arrival in the stacked data. Although XAN exhibits a small amplitude ratio of PKI-IPK with respect to PKIKP in Figure 3D, nonetheless the slant-stack of 15 earthquakes (Fig. 4C) produces a significant arrival with the timing of PKI_{100}-IPK and approximating the ray parameter of PKIKP. However, slant stacking the ENH events (not shown) showed no additional trend, likely due in part to the narrow distance aperture (179.25°–178.9°) of the 9 antipodal earthquakes. Model fits to the neighboring Chinese stations—QIZ, ENH, XAN—antipodal to Chilean earthquakes are shown in Fig. 8. In modeling the low noise data for individual earthquake–receiver pairs, we see in Fig. 8 some waveform evidence of PKI_{100}-IPK at ENH.
3.2. $V_P$ and $V_S$ considerations

The “$V_P$” model series (Supplement Figs. S8 and S9) did not change the inner core shear velocity from PREM ($V_S = 3.5$ km/s), but to fit the phase $PKI_{100}$, $IKP$ we found it necessary to increase $V_S \geq 5$ km/s. Four shear wave models with $V_P$ unchanged from PREM are shown in Fig. 6.

Models Vs1 and Vs2 differ with liquid and solid (PREM) values, respectively, above 100 km depth. Models Vs3 and Vs4 are derived from Vs1 and Vs2 by setting an apparent discontinuity at 250 km depth—transitioning between the $V_S \geq 5$ km/s and a PREM $V_S$ central core—to match $PKI_{250}$, $IKP$ in Fig. 7.

By incorporating the shear velocity influence upon the reflection in $PKI_{100}$, $IKP$ we have not assumed $a$ priori a value for Poisson’s ratio $\nu$, expressed in terms of $V_P$ and $V_S$ as

$$\nu = \frac{V_P^2 - 2V_S^2}{2(V_P^2 - V_S^2)} \quad (2)$$

Within the inner core of the PREM $\nu = 0.44$, where the high value reflects the low shear wave velocity. In the PREM mantle, Poisson’s ratio has a mean $= 0.29$, close to a Poisson solid (0.25) wherein $V_P = \sqrt[3]{3} V_S$. For a high shear wave velocity structure $>100$ km below the ICB, $V_S \sim 5$ or $\sim 6$ km/s, the implied Poisson’s ratios are 0.37 or 0.29, respectively, below the value $\nu \cong 0.5$ for a liquid. Note that a Poisson’s ratio of 0.37 compares well with Birch’s, 1952 estimate, whereas the 0.29 ratio approximates the PREM mantle values.

Mineral physics provides no specific guidance on $\nu$ in the inner core, and the relationship between changes in $V_P$ with those of $V_S$. Waveform models of inner core structures have modified PREM and AK135 $V_P$ without apparently changing $V_S$ (e.g., Song and Helmberger, 1998; Cormier and Attanayake, 2013). Mineral physics experiments and extrapolations do observe higher shear velocities for some candidate inner core compositions (Fig. S11).

There is no specific constraint upon $V_P$ in our data other than timing. Changing the $V_P$ also changes the timing of “PKI” and “IKP” path segments traversing the uppermost inner core. The waveform data (T $\sim 10$ s) of Cormier and Attanayake (2013) are modeled as a $\sim 1\%$ $V_P$ decrease at the ICB and then increasing $V_P + 1\%$ to PREM $V_P$ values near 140 km depth beneath the ICB located under the Atlantic Ocean. We have tested a $V_P$ model based upon the Cormier and Attanayake (2013), modifying the depth of their $1\%$ $V_P$ change to 100 km. Increasing $V_P$ by $+1\%$ to PREM $V_P$ at 100 km depth shifts the $PKI_{100}$, $IKP$ time by about 0.8 s later. To re-align the timing, it is necessary to change the interface depth...
models (Vs3 and Vs4, Fig. 6). The traces are aligned on matching the timing of computed and stacked for the PREM model (Fig. S8). The lower two traces—PKIKP—of the antipodal distance vertically-propagating, near-source shear energy to the antipodal compressions from Fig. 3. For PKIKP “bull’s eye” target (Butler and Tsuboi, 2010). The two largest amplitudes occur near the antipode (Δ = 179.71 and 179.43°) at the peak. At Δ = 179.15°, the PKIKP amplitude significantly reduces, as the distance range crosses a zero of the underlying Bessel function J0. By Δ = 178.9° the amplitude rises again, then approaches another zero of Bessel J0 at Δ = 178.24°. The observed amplitude of PKIKP, which was modeled for Vs = 5.0 km/s at the top of the central core, meets the specific form of amplitude dependence near the antipode.

4.2. Reflection great circles

The location of the reflection great circle for TAM is shown in yellow in Figs. 10 and 11 where PKIKP reflects from the ICB and PKIKP reflects from the apparent discontinuity (Vs ≥ 5 km/s) at 100 km depth below the ICB. The small green circle at Hawaii shows the scaled, inner-core P wavelength J0 projected upwards to the Earth surface. TAM shows little overlap with the nearly orthogonal coverage by QIZ (blue), ENH (white), and XAN (red), whose great circle paths differ by no more than 6 J0 between QIZ and XAN. For PTGA (orange) which shares the observations of PKIKP with TAM, the reflection great circle is within about Δ = 20° of the QIZ, ENH, and XAN neighborhoods. The
antipodal diameter for PTGA is rotated 20.7° from QIZ, which itself is rotated 14.7° from XAN. At the ICB the P wavelength \( \lambda_P \) is \( \sim 55 \) km. The arcuate, inner-core distance between PTGA and QIZ is \( \sim 8 \lambda_P \), whereas between QIZ and XAN this distance is \( \sim 5.5 \lambda_P \). These arcuate distances are also the maximum arcuate distance between the reflection great circles, e.g., the reflection arc (PKI\textsubscript{100},IKP) for PTGA is less than \( <8 \lambda_P \) from QIZ at the ICB. Of the PTGA great circle, only about 22.5° near Southern Africa (Fig. 11A) and 22.5° near the center of the Pacific (Fig. 11B) are within a wavelength of QIZ or XAN. Hence, 315° of the PTGA great circle lies more than a wavelength from the QIZ, ENH, XAN great circles.

For TAM the overlap with QIZ, ENH, and XAN is essentially two wavelengths as the two cross-over arcs with TAM are nearly orthogonal.

Finally, Fig. 11C,D shows the coverage of PKI\textsubscript{100},IKP for TAM and PTGA where broadly differing propagation paths display very similar characteristics. Hence, common inner core structures include not only proximal stations (QIZ, ENH, and XAN), but also broadly distinct propagation surfaces (TAM and PTGA). Comparing the narrow, great-circle reflection coverage of the QIZ, ENH, and XAN with the wider, great-circle reflection coverage of the TAM and PTGA suggests that the latter coverage of the inner core is more extensive than the former and therefore may be more representative of the inner core.

4.3. Shear velocity estimates and inner core composition

The primary influence on the amplitude of PKI\textsubscript{100},IKP is the shear
wave velocity contrast across an apparent discontinuity near 100 km depth (Fig. 6). We have considered a liquid with \( V_S \sim 0 \) above this interface for TAM and PTGA data. To achieve a good match between data and 3DSEM synthetic, we found that the shear wave velocity below this interface is consistent with \( V_S \geq 5 \). We note that \( PKI_{100} \) IPK by itself does not constrain the inner core beyond a wavelength (~55 km) below the interface. Models Vs1 and Vs3 extend \( V_S = 5 \) to the center of the inner core. However, the observation of \( PKI_2\)IKP interpreted as \( PKI_{250} \) IKP suggests an interface at 250 km depth (150 km thick with \( V_S \geq 5 \), Fig. 6, Vs3 and Vs4). The sole shear wave velocity constraint upon the deeper core arises from consistency of normal modes with PREM. To acknowledge this constraint, we have assumed PREM velocities below the reflection depth of \( PKI_{250} \) IKP (Fig. 6).

The list of \( V_S \) estimates from normal mode observations and models, as well as \( PKI_{250} \) (a shear wave traversing the inner core), in Table S2 has been focused at 3.5 km/s—near the starting value assumed when normal modes became a significant contributor to Earth structure. Since a 3.5 km/s shear velocity is slower than extrapolated for an iron-nickel composition, mineral physicists have explored the effect of lighter alloyed elements (Hirose et al., 2013)—e.g., carbon (Chen et al., 2014) and phosphorus (Lai et al., 2020)—on the shear velocity of the inner core. Nonetheless, shear velocities reported from \textit{ab initio} calculations (Vocadlo, 2007; Deuss, 2008) and experimental results (Mao et al., 1998) for iron and its alloys at inner core pressure and temperature can reach \( V_S > 6 \) km/s (Fig. S11). However, mineral physics experiments may also discount additional effects (e.g., Attanayake et al., 2018) such as pre-melting near a homologous temperature of ~1 (Martorell et al., 2013) and inaccuracies of linear extrapolation used to determine seismic properties at inner core pressure and temperature conditions (Martorell et al., 2016).

4.4. 3D structures

The inner core structures proposed—though inherently non-unique—provide a simple framework to match the observations of \( PKI_{100} \) IKP and \( PKI_{250} \) IKP, which can be further improved. The broad dichotomy between the antipodal propagation paths for TAM and PTGA versus the three neighbors—QIZ, ENH, and XAN—suggests large-scale heterogeneity of the inner core shear wave velocity. The TAM and PTGA data suggest large portions of the uppermost inner core are manifest as a liquid/mushy region down to 100 km below the traditional inner core boundary.

For QIZ, ENH, and XAN the uppermost inner core appears “PREM-like” at 3DSEM periods \( T > 4 \) s and may be consistent with observed high-frequency \( PKIKP \) (1 Hz) reflections (Deuss, 2008; Leyton and Koper, 2007). However, TAM and PTGA propagation paths effectively resolve a solid-liquid (or mushy) interface ~100 km below the ICB. Since \( PKIKP \) is not prominent in the seismogram, it is difficult to resolve structures just below the ICB. Therein the differing frequency content between \( PKI_{100} \) IKP and \( PKIKP \) observations may suggest a frequency dependence of the interface properties—e.g., \( PKIKP \) with a wavelength \( \lambda_P \) of ~11 km responds to smaller structures than \( PKI_{100} \) IKP with a wavelength \( \lambda_P \) of ~55 km. In the Fig. S13, we plot the \( PKIKP \) reflection arcs for TAM, PTGA, and ENH with respect to \( PKIKP \) reflection points (Waszek et al., 2011). Although there is some semblance of correspondence between regions without \( PKIKP \) observations (whether noisy or missing) and where \( PKI_{100} \) IKP propagates, we have approached colleagues to intercompare \( PKIKP \) reflection points with \( PKI_{100} \) IKP reflection arcs. Finding common junctures between \( PKIKP \) reflection arcs and \( PKIKP \) reflection points may help to clarify this detail of the ICB. Finally, we have not emphasized the fit to the \( PKIKP \) arrival (see Butler and Tsuboi, 2010; Tsuboi and Butler, 2020) which follows \( PKI_{100} \) IKP—though both are synthesized together. To tackle the issues of \( PKIKP \) and \( PKI_{100} \) IKP we will have to resort to 3DSEM synthetics complete to \( T < 2 \) s.

Considering only \( PKI_{100} \) IKP, there is no resolution of the bottom of the high, shear-wave velocity region which could in principle extend to the center of the core. Nonetheless, the observational modeling of \( PKI_{250} \) IKP effectively resolves this question: the region is ~150 km thick and extends to ~250 km depth. In choosing a PREM shear wave velocity below this apparent discontinuity, we acknowledge the research in Table S2, but have no direct constraint ourselves.

With the current data set we cannot effectively resolve the existence, thickness, or velocity of a possible discontinuity at 100 km depth associated the propagation paths to QIZ, ENH, and XAN. Nevertheless, models with solid PREM velocities above 100 km depth fit the Chinese station data better than a mush/liquid (e.g., Vs1). Nonetheless, there is suggestive evidence of a solid–solid interface (model Vs2) in the ENH antipodal waveforms and XAN slant stacked data (Figs. 5 and 8).

4.5. Normal modes

Using the PREM (no ocean) model as reference, we have calculated normal modes for \( PKIKP \) (sensitive to variations in inner core \( V_S \) and \( PKIKP \) modes (sensitive to variations in inner core \( V_P \) and \( V_S \) discussed in (Deuss, 2008; Robson and Romanowicz, 2019), using the Mineos code.
Table S3 compares a PREM model (without ocean) with a modified PREM model substituting a high shear wave velocity ($V_S = 5$ km/s) between radii $r = 992$ and 1107 km throughout the inner core. This preliminary comparison of normal modes with PREM shows little resolution of the proposed 3D inner core structures, as simplified for Mineos to a 1D model. This 1D model matches the modal frequencies of PREM (no ocean) considered within $< 0.1\%$ for 13 of 19 modes for $V_S = 5$, where the remainder of the modes tested fit within $< 0.4\%$, respectively. Hence, the normal modes tested are not strongly sensitive to $V_S = 5$ km/s layer girding the inner core at a depth of 114–230 km. We should note, however, that the order of eigenfrequency shift for PKJKP and PKIKP modes discussed here should be considered as an upper limit because we have introduced the $V_S = 5$ km/s layer over the entire inner core at a depth of 114–230 km, rather than apportioned along the great circles circumscribing the antipodal propagation paths for TAM and PTGA. We propose that since this feature is not a global inner core structure, the resultant eigenfrequency shifts should actually be much less than our modeling has suggested.

4.6. Mushy uppermost inner core

In addition to a liquid/solid interface, we also considered a dendritic, mushy zone with a low shear wave velocity (Fearn et al., 1981). Sumita (2010) notes, “The mushy zone is the mixed solid–fluid region during dendritic growth. Mushy zones are ubiquitous features in directionally solidifying metallic alloys...”. Deguen et al. (2007) note, “The thermodynamic thickness of the resulting mushy zone can be significant, from ~100 km to the entire inner core radius, depending on the phase diagram of the core mixture.” Nonetheless, Deguen et al. (2007) predict that such a thick mushy zone would collapse under its own weight, on a much smaller length scale (~1 km), but caution, “The length scale of the thermodynamic depth of the mushy zone and the compaction length scale, although clearly different, are unconstrained by the current knowledge of the phase diagram of the core mixture and of the solid iron viscosity at inner core conditions.” Alexandrov and Malygin (2011) derived the main features of inner core solidification scenarios with two simple nonlinear models of the slurry and mushy layers in the regions, where fluid is rising or descending, and conclude that the thickness of the phase transition layer (slurry or mushy) near the ICB is of the order of a few hundreds of meters. Finally, Lasbleis et al. (2020) conclude “… the amount of melt trapped in the bulk of the inner core is a record of the
Krasnoshchekov et al. (2005) have observed in nuclear explosion data from central Asia a “mosaic” pattern of pre-critical PKiKP reflections from the top of an inner core with solid and mushy patches. These paths reflect on or near TAM and PTGA paths highlighted in Asia (Figs. 10 and 11), where a mushy zone at the ICB is hypothesized.

In the supplement (Fig. S14) we compare synthetics for liquid ($V_S = 0$ km/s) and mushy ($V_S = 0$ to 0.3 km/s gradient) layers above an interface boundary with a PREM solid ($V_S \sim 3.5$ km/s) at a radius of 1100 km (models Vp4 and Vp4.4, Figs. S8–S9). We see in Fig. S14 that compared with the liquid, a mushy region's effect on amplitude and timing is negligible—showing that the models cannot effectively resolve features of a low-velocity $V_S$ mush region gradient from a liquid region. Since we do not have a strong constraint on the shear wave velocity for a mushy region, we consider $V_S = 0$ as one end member for further 3DSEM calculations, recognizing that a mushy zone with shear velocity $\gtrapprox 0$ km/s may also fit the data.

### 4.7. Quasi-hemispheric uppermost inner core

Significant lateral heterogeneity has been observed at the top of the inner core (Tanaka and Hamaguchi, 1997; Niu and Wen, 2001; Wen and Niu, 2002; Stroujkova and Cormier, 2004; Attanayake et al., 2014; Waszek and Deuss, 2011; Garcia et al., 2006). Each of these studies have observed faster PKiKP velocities (relative to a reference phase, either PKiKP or PKPbc) in the quasi-eastern hemisphere ($qE \sim 40^\circ$E to $180^\circ$E) than the quasi-western hemisphere ($qW \sim 180^\circ$W to $40^\circ$E). Fig. 10 also plots the proposed boundaries of $qE$ and $qW$ regions, which extend latitudinally from pole to pole. These PKiKP paths propagate with the same turning points as PKIIKP path segments (~87 km depth), and hence propagate predominately above the central core. Anisotropic velocities (Waszek and Deuss, 2011) in $qE$ are ~0.8% faster than in $qW$. As observed in Figs. 10 and S13, the $PKI_{100}$ propagation surface for TAM samples both $qE$ and $qW$, but travels below 100 km depth within the central core. Hence there is not a simple mapping of our antipodal coverage onto the $qE$ and $qW$ regions advocated.

Following Section 4.5 we have also considered an “analog” model with a hemispheric dichotomy, which also considers an uppermost inner core with contrasting liquid and solid features in a hemispheric context. This employs a fictitious uppermost layer of low shear wave velocity ($V_S = 1.75$ km/s), which averages $V_S = 0$ liquid and $V_S = $ PREM solid at the top of the Inner Core between radii $r = 1121.5$ and $1221.5$ km. In comparison with PREM (no ocean) in Table S4, the modified PREM ($V_S = 1.75$ km/s) has 11 of 13 modes within $<0.1\%$, and 2 within $<0.2\%$. Hence, compared with the PREM (no ocean), the normal modes tested are relatively insensitive to a mean $V_S = 1.75$ km/s at the top of the inner core.

Finally, whereas in a broad sense, although much of the ICB has been sampled by PKiKP, the reflection area of each sample (1 Hz at 11 km/s ~

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**Fig. 11.** The great circles show the projection (AB) to the Earth’s surface of the PKiKP and PKI$_{250}$IKP midpol reflection, great-circle arcs within the upper core. The colored labels correspond to the observing station’s reflection arc. The lower projection (CD) follows the format of Fig. 10. The small green circle in (A) centered on Hawai’i Island has a projected diameter of 1 wavelength on the ICB. TAM (yellow) and PTGA (orange) both observe PKI$_{100}$IKP and PKI$_{250}$IKP, though propagating over substantially differing paths. For the neighboring paths of XAN, ENH, and QIZ (red, white, blue, respectively) within the upper central core, evidence for PKI$_{100}$IKP reflecting from a mush/liquid–solid interface at 100 km is absent, though a solid-solid interface cannot be rejected in every respect. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
380 km²) represents only 0.002% of the surface of the inner core. Therefore, even the extensive study PKIKP by Waszek and Deuss (2011) which measured 2615 seismograms sampled only ~5% of the ICB, assuming no overlap of samples.

5. Summary

In this study of the inner core of Earth we have presented antipodally amplified arrivals between PKIKP and PKIKP. Two sets of data are presented—TAM (Algeria) and PTGA (Brazil) both show clear seismic arrivals, whereas the Chinese stations (XAN, ENH, QIZ) do not—not indicating significant heterogeneity within the upper inner core. Where observed, the precursors arrive 7 and 17 s before PKIKP, and the former precursor is amplified greater than PKIKP and > 50% of PKIKP. The characteristics of the precursors are modeled using the 3D Spectral Element method. More than 16 series of inner core models were tested, including changes from PREM $V_p$, $V_S$, and density $\rho$ (Figs. 5–9, S8, S9, and S14).

The TAM and PTGA were modeled with the same structures, changing only the global Centroid Moment Tensor earthquake source mechanism. The immediate precursor to PKIKP is cast as an analog phase, $PKIKP_{100}$, which reflects from an apparent discontinuity ~100 km below the ICB. Slant-stack measurements indicate the $PKIKP_{100}$ ray parameter ($p \approx 1.8$ s⁻¹) approximates that of PKIKP. A principal challenge is to match the $PKIKP_{100}$ PKIKP arrival amplitude, which exceeds that of PKIKP. The only model found meeting this constraint approximates a $V_S$ solid-liquid discontinuity where $V_S \geq 5$ for the solid (Fig. 5).

However, the waveform cannot resolve differences between liquid and a “mushy” region. Including a 1% change in $V_p$ at this $V_S$ interface, the apparent depth of the interface increases by 20 km (Fig. 9). The earlier (17 s) precursor to PKIKP may be matched ($PKIKP_{1250}$) assuming an apparent discontinuity (solid-solid) at ~250 km depth below the ICB. (Fig. 7).

The reflection surfaces in Fig. 11 are grouped. TAM and PTGA data conform with a 100 km thick $V_S \sim 0$ liquid/mush region overlaying high velocity ($V_S \geq 5$ km/s). For the Chinese stations (XAN, ENH, QIZ) antipodal to Chilean earthquake, the situation is quite different. The stacked amplitude-phase data show little evidence of PKIKP precursors, except XAN. The slant stacked data for XAN shows a clear $PKIKP_{100}$ PKIKP arrival with a ray parameter approaching PKIKP. A PREM outer core (+3D mantle) structure fits the Chinese sites better than the models which fit TAM and PTGA (Figs. 5, 7, 8, 9). Nonetheless, a solid-solid interface (PREM $V_S \geq 5$ km/s) at 100 km depth ($V_S \geq 5$) fits the data (Fig. 8E), where QIZ is not well resolved. The XAN and ENH may conform with a 100 km PREM region overlying high velocity ($V_S \geq 5$).

Prior inner-core, body-wave research has utilized PKIKP, PKIKP, PKIKP, PKIKPd, and PKIKPbc phases to constrain the inner-core compositional wave velocity. The heterogeneity observed includes regions of $P$-wave anisotropy and isotropic velocity variations of <5% which differ with depth and hemisphere (~1%). In contrast the heterogeneity in shear wave velocities is larger and encompasses the inner core with distinct mushy/liquid and solid regions. Such features may lack structural details observable by PKIKP at the wavelengths interrogated, $\lambda > 55$ km. Fig. 11 suggests that the top of the inner core is manifested as regions of liquid/mush and regions of PREM solid.

Normal modes provide constraints on inner core shear wave velocities (Table S2). We have tested the modal fits for PKIKP and PKIKP modes (Tables S3 and S4), comparing PREM (no ocean) with two example model modifications. Both PREM modifications showed close fits—most within <0.1%—to normal modes of the basic PREM (no ocean) model. Nonetheless, models tested were homogeneous within the inner core, whereas the antipodal data are clearly reflect a heterogeneous, inner core structure.

Alternate hypotheses can be considered. The principal alternative to PKIKP—with an apparent discontinuity at 100 km depth due to a mush discontinuity over a high shear wave velocity ($V_S \geq 5$) substrate—is $PKIKP_{100}$ due to a solid-solid discontinuity at 100 km depth. Note that nomenclature would be identical since the arrival is marked by the interface depth. The challenge is to match the antipodal data with a $V_p$ velocity change that still fits prior $V_p$ data sets, e.g., PKPBc-PKIKP, and PKIKP-PKIKP. A clear observation of $PKIKP_{100}$, a top-side reflection from the 100 km depth interface, could provide an additional constraint on the travel time in the mush region, and reflection coefficient at the bottom. Near vertical PKIKP reflections from the inner core above central Asia are reported with a mosaic of solid and liquid patches over distances of 23–240 km. These underly the TAM and PTGA PKIKP reflection great circles beneath Asia. Since the $PKIKP_{100}$ reflection great circles (Fig. 11) overlap at $T > 4$ s with regions of the inner core parsed as quasi-eastern and quasi-western provinces at $T \sim 1$ s, common sample locations at the IOCB could help to determine the frequency dependence of such features. In order to model these short period features by 3DSEM, an octave greater frequency resolution is required.

The PREM model has guided mineral physicists in their quest for understanding the low shear wave velocity within the inner core. Nonetheless, we now see body-wave reflection evidence for shear wave velocities $\geq 5$ km/s from an upper core interface that comports with high shear wave velocity, iron-nickel alloyed compositions. Whereas we have antipodally interrogated the inner core with compositional body waves, our results are specifically derived from measured reflection amplitudes and times dependent upon shear wave velocity contrasts at apparent discontinuities 100–250 km beneath the traditional inner core boundary. Where a discontinuity is not observed, the inner core structure more closely approximates PREM.

Acknowledgments and data availability

This work is partially supported by JSPS KAKENHI Grant Number JP21K03710. Data were obtained from GEOSCOPE and the IRIS Data Management System. We used the computer program (SPECFEM3D) for Spectral-Element Method. All the computations are performed using the Earth Simulator at the Earth Simulator Center of JAMSTEC. Centroid moment tensor solutions (GCMT) are used for synthetic models. We used the Mineos code distributed by the Computational Infrastructure for Geodynamics for computing eigenfrequencies for inner core modes. We thank GEOSCOPE, USGS and NSF, and NCDSN China for the operation and maintenance of the seismic station used in this study. All data were downloaded from the IRIS Data Management System. Earthquake parametric data were downloaded from the USGS earthquake catalog and tabulated in the Supplementary Information file. Earthquake source mechanisms were downloaded from the Global Centroid Moment Tensor database. The authors thank the two anonymous reviewers for their comprehensive reviews. The authors would like to thank SOEST contribution number 11397 and HIGP contribution number 2449.

Declaration of Competing Interest

There are no conflicts of interest to declare.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.pepi.2021.106802.

References


